A STUDY OF THE TORSIONAL RIGIDITY OF TWO-TRACK RAILROAD BRIDGES USING A CELLULOID MODEL WITH LOADING ON ONE TRACK ONLY

BY

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SECTION I

(a) Introduction, (b) Objectives, (c) Procedure

(a). Extensive research by the writer has failed to disclose any definite instance of an exhaustive investigation, in this country, of the problem of the torsional stiffness or rigidity in two-track railroad bridges, when loaded on one track only. In one case, (Sciotoville Bridge) brief reference[1] was found concerning this question, but no detailed solution was given, although transverse bracing of a rather unusual type was included in the design of the bridge. Data were not available concerning the method of design of this bracing although it is presumed that some form of tenable analysis was used.

Included in certain writings[2] by Professor F. P. Witmer, of the Towne Scientific School, is a brief discussion of the problem of torsional rigidity in the case of the double-track ferry bridge, when loaded on one track only. In this case, it is pointed out by Professor Witmer that the problem is of considerable importance, several failures in the floor beam connections to the trusses having occurred in bridges of this type. The greater importance of torsional rigidity in ferry bridges is, of course, due to the fact that one end of the span rests on

(1) Transactions, A.S.C.E., Vol.No.85,1922.
(2) Advanced Structures Notes on Movable Bridges by Professor F.P.Witmer, Director, Civil Engineering Dept., Towne Scientific School, University of Pennsylvania.

a yielding support (a water borne pontoon). The solution in this type of construction has been to make very rigid floor beam connections and design the chords and diagonals so that the trusses may twist with some freedom, the result being that the plane of each transverse frame will rotate about an axis parallel to the bridge axis, each successive plane away from the shore rotating to a slightly greater degree than the preceding one, the accumulated rotations representing the total twist.

While this specific problem is not within the scope of this investigation, it is pointed out that the model is readily adaptable for such a study, and from a consideration of the results given hereinafter, it is believed apparent that valuable and informative data might be secured.

The writer recognized that the scarcity of published analyses concerning the torsional stiffness of railroad bridges finds justification in the fact that wind load analyses, practical design, and construction considerations are the usual criteria for the lateral system. It was believed, however, that a theoretical study of this bridge characteristic (torsional rigidity), together with a series of experiments on a model might prove to be interesting, if not definitely instructive, and might arouse further academic questions on the subject. The use of a model, in particular, was considered to be especially desirable, in view of the indeterminate nature of the problem. In order that the model results may be considered reliable, both

in this and in future experiments, it was believed necessary to devote some effort to verifying the calculated deflection characteristics of the model by experimental tests.

Additional interest in the subject of torsional rigidity was aroused by the discovery that Dr. R. K. Bernhard, formerly with the German State Railways, had available some field data concerning measured joint deflections under single track loading of several bridges secured during a study of bridge torsion, which he made in Germany. It is with great appreciation that the writer includes, hereinafter, these data supplied by Dr. Bernhard.

(b) The object of this investigation was to study the torsional stiffness of a celluloid model bridge and to correlate the results with a theoretical analysis. A secondary objective was to determine the feasibility of model studies for application to an indeterminate structural problem with the attainment of results which might be confirmed by comparison with actual field data. Insofar as possible, it was desired to determine other incidental structural characteristics which could be noted from the data secured in the torsion tests.

(c) The procedure followed in the actual investigation was similar to the order of subject matter presented herein. In Section II a theoretical analysis of the problem is given. There is then introduced, in Section III, a torsion stiffness factor, "k", which is used as a measure of the relative torsional rigidity of a span. The derivation and definition of "k" are given, as

well as "k" values for the bridges tested in Germany.

A complete consideration of the model design is then presented in Section IV, and in Section V, theoretical and experimental structural characteristics of the model are compared.

The application of the torsion stiffness theory to the model is given in Section VI, and Section VII contains the test procedure and results for the celluloid model and a brief discussion of the elastic curves plotted from test results.

Section VIII contains a discussion of the test results and a comparison with theoretically predicted results. Suggestions for further study are made and reference is made to field deflections in comparison with the model deflections.

SECTION II

Theoretical Considerations

The following discussion is at first confined to the center frame of the bridge. This frame is considered as an isolated structure composed of two vertical and two horizontal members, the former being the verticals of the trusses, the latter the bridge floor beam and the tie between opposite upper panel points. The frame is supported as shown in Fig. 1, by two vertical struts which represent the support afforded by the main trusses, and two horizontal struts which represent the support afforded by the horizontal trusses. The struts are elastic and their lengthening or shortening represents the deflections of the main and wind trusses. If we neglect the bending of the members of the frame, we can consider this structure as a rigid body supported as shown. It is apparent that this consideration results in one external redundant (support at C). An examination of the frame itself shows that there are two internal redundants (the rigid connections at q and r). In the development of this analysis we shall consider joints q and r to be hinged. This assumption can be made without serious error when the actual upper ties are light sections, as where the joint

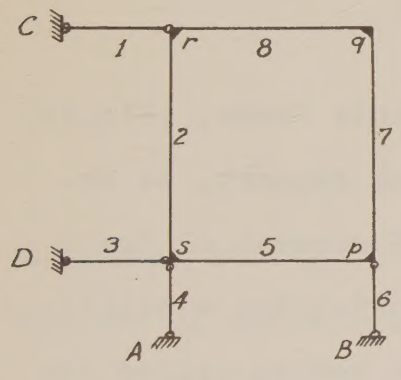


Figure 1

is not especially designed to take moment. The internal redundants then disappear.

If we load the frame with an eccentric load P (Fig. 2), taking moments of external forces about p, and letting $\frac{h}{b} = n$:

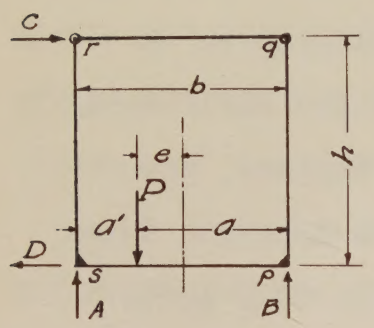


Figure 2

$$\Sigma M_p = Ab - Pa + Ch = 0$$

$$A = \frac{Pa}{b} - \frac{Ch}{b}$$

$$A = \frac{Pa}{b} - Cn \qquad (1)$$

Also:

$$\Sigma M_s = Ch + Pa' - Bb = 0$$

$$B = \frac{Pa'}{b} + Cn \qquad (2)$$

$$\Sigma V = A + B - P = 0$$

$$A = B - P \qquad (3)$$

$$\Sigma H = C + D = 0$$

$$C = -D \qquad (4)$$

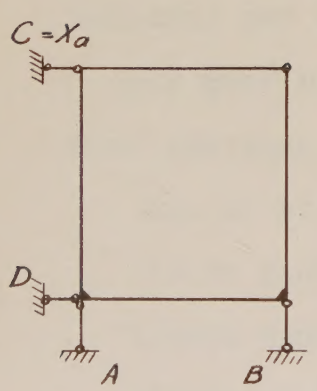


Figure 3

The solution of this frame, (Fig.3), having a redundant support, is accomplished by the removal of the redundant, thus obtaining a statically determinate system called the "principal system."[3] This redundant, together with the external loads, is then reapplied on the principal system which, being statically determinate, can be solved.

(3) Structural Engineering Problems by David A. Molitor. Reproduced by the Bureau of Yards and Docks, Navy Dept. 1937.

To find the redundant reaction X_a we shall use an application of Mohr's work equation combined with Maxwell's Law (page 23 of reference /3/).

This principle can be expressed by the equation:

$$\delta_a = P_m \delta_{ma} - X_a \delta_{aa} \qquad (5)$$

where: δ_a = the displacement of the point of application of the redundant X_a in the direction of X_a due to loads P_m and X_a

P_m = the external load applied at point m

δ_{ma} = the displacement of m due to the application on a system of only the load $X_a = 1$

X_a = the redundant force, in this case a reaction

δ_{aa} = the displacement of the point of application, a, of the redundant X_a in the direction X_a when the system is loaded with $X_a = 1$.

Referring to Fig. 1 we see that δ_a being the displacement at the end of the strut, is 0, then equation (5) becomes:

$$X_a = \frac{P_m \delta_{ma}}{\delta_{aa}} \qquad (6)$$

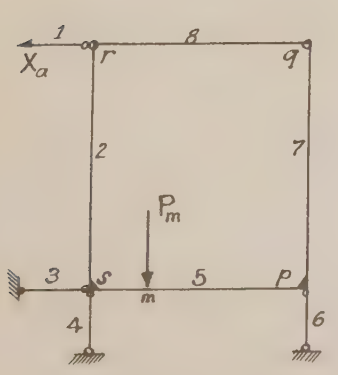


Figure 4

Fig. 4 shows the principal system acted on by load P_m and the redundant X_a. When only $X_a = 1$ is acting, the displacement of point m in the direction P_m depends only on the vertical movement of joints s and p which in turn depend on the stress in members 4 and 6.

From equation (1), the total stress in member 4

$$= -X_a n = -n \qquad (7)$$

From equation (2), the total stress in member 6

$$= +X_a n = +n \qquad (8)$$

Let ℓ_i and A_i be the length and cross-sectional area of any member i, and E be the modulus of elasticity. Then, from Hooke's Law:

$$\Delta_s = \frac{(\ell_4)(-n)}{A_4 E} \quad \text{and} \quad \Delta_p = \frac{(\ell_6)(n)}{A_6 E}$$

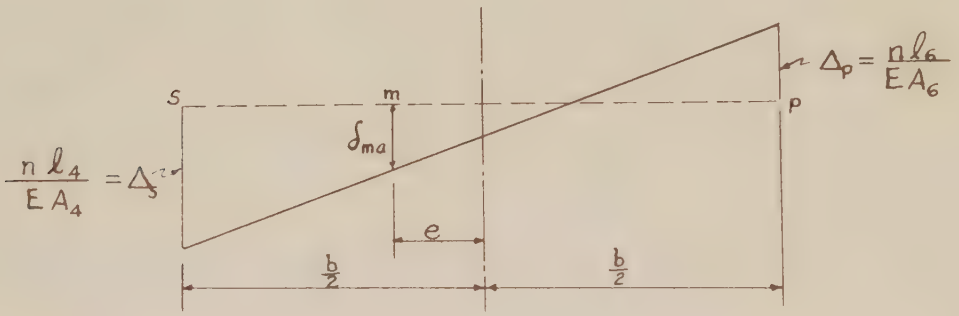


Figure 5

From Fig. 5 —

$$\delta_{ma} = \frac{b(\Delta_s - \Delta_p) + 2e(\Delta_s + \Delta_p)}{2b} \qquad (9)$$

Now to find δ_{aa} the principal system is again loaded with $X_a = 1$ and we find the displacement at a due to bending and direct stresses of all members of the system. Mohr's work equation[3] for this displacement is

$$\delta_{aa} = \frac{1}{EA}\int_0^{\ell} N^2 dx + \frac{1}{EI}\int_0^{\ell} M^2 dx \qquad (10)$$

where the letters have the customary significance.

The first term on the right hand side of this expression indicates the displacement due to strain in members having direct stress. The second term expresses the displacement arising from members subjected to bending. Neglecting the displacement due to direct stresses in members 2, 5, 7 and 8 which are relatively insignificant, we evaluate the displacement of point a due to the direct stress in members 1, 3, 4 and 6 (stresses from equations (3), (4), (7) and (8)):

$$\frac{1}{EA}\int_0^{\ell} N^2 dx = \frac{1}{E}\sum N^2 \frac{\ell}{A} = \frac{1}{E}\left(\frac{n^2\ell_6}{A_6} + \frac{n^2\ell_4}{A_4} + \frac{\ell_1}{A_1} + \frac{\ell_3}{A_3}\right) \qquad (11)$$

To evaluate the second term on the right hand side of equation (10) we assume:

a) the redundant $X_a = 1$ is distributed equally by member 8 into members 2 and 7 (members 7 and 8 being identical.

b) the moment in member 5 at joint s (also at p) is not the same in magnitude as the moment in member 2 since there is additional restraining moment due to the diagonals connecting at this panel point.

c) the displacement δ_{aa} is twice the displacement integrated from point a to the center of member 5, that is,

$$\frac{1}{EI}\int_0^{\ell} M^2 dx = \frac{2}{E}\left[\int_0^{h} \frac{M^2}{I_2} dx + \int_0^{\frac{b}{2}} \frac{M^2}{I_5} dx\right]$$

This is apparent from the symmetry of the frame

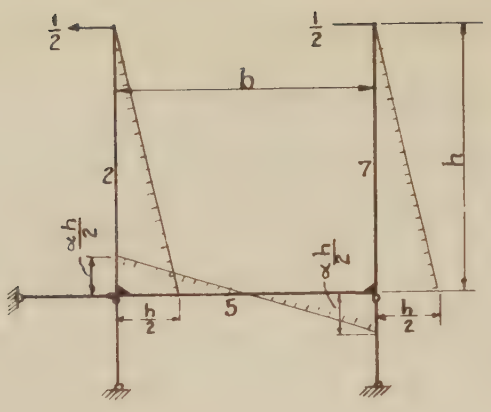


Figure 6

The moment curves for the frame are as shown in Fig. 6. The factor α used in the moment in member 5 is in accordance with (b) above and will be discussed hereinafter. The displacement of a due to bending stresses becomes:

$$\frac{1}{EI}\int_0^{\ell} M^2 dx = \frac{2}{E}\left[\frac{1}{I_2}\int_0^{h}\frac{1}{2}x^2 dx + \frac{\alpha^2 h^2}{I_5}\int\left(\frac{1}{2}-\frac{x}{b}\right)^2 dx\right]$$

$$= \frac{2}{E}\left[\frac{1}{12 I_2}h^3 + \frac{\alpha^2 h^2}{I_5}\left(\frac{b}{24}\right)\right] \qquad (12)$$

Then $$\delta_{aa} = \frac{1}{E}\left(\frac{n^2 \ell_6}{A_6} + \frac{n^2 \ell_4}{A_4} + \frac{\ell_1}{A_1} + \frac{\ell_3}{A_3} + \frac{h^3}{6 I_2} + \frac{\alpha^2 h^2 b}{12 I_5}\right) \qquad (13)$$

Equation (6) becomes (from (9) and (13)):

$$X_a = EP_m \cdot \frac{b(\Delta_s - \Delta_p) + 2e(\Delta_s + \Delta_p)}{2b\left(\frac{n^2 \ell_6}{A_6} + \frac{n^2 \ell_4}{A_4} + \frac{\ell_1}{A_1} + \frac{\ell_3}{A_3} + \frac{h^3}{6 I_2} + \frac{\alpha^2 h^2 b}{12 I_5}\right)} \qquad (14)$$

When the main trusses are symmetrical, the quantities Δ_s and Δ_p are equal. This is in effect, the same as saying that members 4 and 6 are identical. For further simplification of equation (14) we will introduce an elastic constant "c" which

may be defined as the normal force necessary to produce unit elongation in a member. From Hooke's law this force is $c = \frac{AE}{\ell}$,

Thus $$\Delta_s = \Delta_p = \frac{n}{C_4} = \frac{n}{C_6}$$

Then $$X_a = P_m \cdot \frac{\frac{2ne}{C_4}}{b\left(\frac{2n^2}{C_4} + \frac{1}{C_1} + \frac{1}{C_3} + \frac{h^3 C_4}{6EI_2} + \frac{\alpha^2 h^2 b}{12 E I_5}\right)}$$

$$= \frac{2 P_m e n}{b} \cdot \frac{1}{2n^2 + \frac{C_4}{C_1} + \frac{C_4}{C_3} + \frac{h^3 C_4}{6EI_2} + \frac{\alpha^2 h^2 C_4 b}{12EI_5}} \qquad (15)$$

It is necessary to interpret the elastic constants of the imaginary struts in terms of the actual trusses. It is obvious that by analogy we may consider the deflection of the truss to be identical in effect to the elongation of the imaginary struts. Thus if 1 pound at the panel point of the truss in question produces a displacement at that point of Δ, then the elastic constant of that truss is $\frac{1}{\Delta}$.

The factor α in the last term of the denominator of equation (15) indicates the ratio of the moment in the floor beam, member 5, to the moment in the vertical at that joint. An exact solution of this factor is impracticable since it would be necessary to consider torsional moment in the lower chord and the stiffness of the diagonals connecting at the joint, as well as the gusset plates. If we neglect the torsional restraint of the chord (which is comparatively small) the restraint at the far ends of the diagonals and floor beam, and the restraint of the

gusset plates, the factor α depends on the $\frac{I}{\ell}$ values of the diagonals and the floor beam.

Let I_9 and s be the moment of inertia and length of the diagonals; let I_5 and b be the same for the floor beam. Then:

$$\frac{2I_9}{s} + \frac{I_5}{b} = \text{sum of stiffness at the joint}$$

$$\alpha = \frac{I_5}{b} \div \left(\frac{2I_9}{s} + \frac{I_5}{b}\right)$$

$$\alpha^2 = \frac{I_5^2 s^2}{(2bI_9 + sI_5)^2}$$

Formula (15) then becomes:

$$X_a = \frac{2Pen}{b} \cdot \frac{1}{2n^2 + \frac{c_4}{c_1} + \frac{c_4}{c_3} + \frac{c_4 h^3}{6EI_2} + \frac{c_4 h^2 b s^2 I_5}{12E(2bI_9 + sI_5)^2}} \qquad (16)$$

The last two terms of the denominator of formula (16) represent the effect on X_a due to the stiffness of the particular frame under consideration. However, in actuality, the behavior of one frame does not determine the bridge stiffness. The effect of all other frames of the bridge must be taken into account. Since the stiffness of the bridge is in direct proportion to the number of frames producing that stiffness, we may multiply the terms referring to the frame by the number of frames acting (two less than the number of panel points), m. Since these terms are in the denominator, we will multiply each by $\frac{1}{m}$. This corrected formula will be referred to as (16A). Application of this formula will be demonstrated in Section VI.

SECTION III

Rigidity Factor and Certain Field Data

In order to arrive at a factor which will express the torsional rigidity or stiffness of a two track bridge, there will be considered now an isolated frame subjected to one sided loading. This frame is supported by elastic struts (Fig. 7) and when loaded asymmetrically will deflect as shown. The vertical components of the reactions at the bottom joints, and therefore the vertical components of the deflections of the bottom joints will be proportional to the distance of P from B and A respectively, thus:

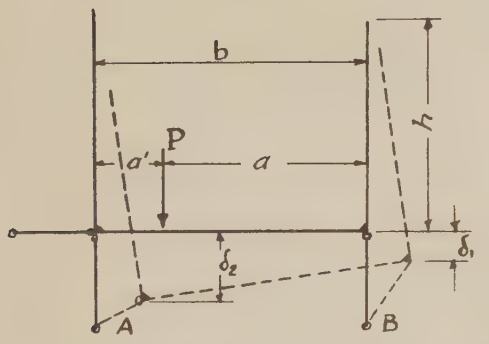


Figure 7

$$\frac{B}{A} = \frac{\delta_1}{\delta_2} = \frac{a'}{a}$$

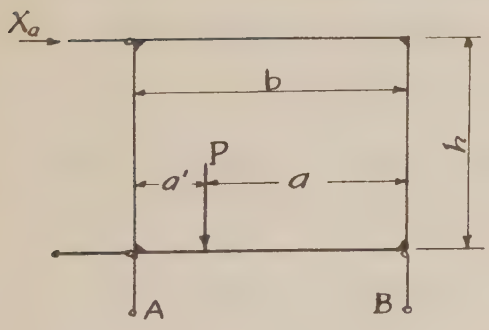


Figure 8

However, as the rigidity of the frame is increased, as, for example, by the addition of an upper horizontal bracing, Fig. 8, it is apparent that there is introduced a restraining moment $X_a h$ (X_a refers to the force discussed in Section II) and the proportionality mentioned above can no longer exist; the reaction at A becomes less and that at B increases. Stated in another way B/A becomes greater

than a'/a or $B/A = \delta_1/\delta_2 = k.(a'/a)$ where k is the "torsional stiffness" factor with a value of more than unity. It is apparent that for the case where no bracing is provided at the top, k = 1. The factor k can be applied to the entire bridge and thus it can be used as a measure of the torsional stiffness of the bridge. Rewriting for the value of k:

$$k = \frac{\delta_1}{\delta_2} \cdot \frac{a}{a'} = \frac{B}{A} \cdot \frac{a}{a'} \qquad (17)$$

But $A + B = P$

Then $$A = \frac{Pa}{a + a'k} \qquad (18)$$

And $$B = \frac{Pa'k}{a + a'k} \qquad (19)$$

With the use of a set of measured deflections for any given bridge under any asymmetrical load, k for that bridge can be determined from equation (17). With this value of k, we can then predict the actual truss loads (at A and B) and from these, the deflections δ_1 *and* δ_2 for any desired load.

It is to be expected that similar bridges will possess like k factors and can be evaluated in the manner shown hereinafter as a reasonable prediction for the torsional rigidity of any proposed bridge.

As noted in Section I, the writer was able to secure test results made on various bridges in Germany. These results are given herewith:

Bridge №	Span (ft.)	Height (ft.)	Width (ft.)	$\frac{a'}{a}$	δ_2 inches	δ_1 inches	$k=\frac{\delta_1}{\delta_2}\cdot\frac{a}{a'}$	Note
1	620	65.5	30.0	.449	.792	.587	1.65	(a)
2	414	65.5	30.0	.449	1.010	.635	1.40	(a)
3	341	34.	29.2	.436	1.261	.828	1.50	(a)
4	290	33.	30.0	.449	.772	.465	1.34	(a)
5	282	21.6	27.2	.407	.954	.626	1.61	(b)
6	150	22.4	28.4	.407	.224	.099	1.08	(a)
7	148	18.4	30.0	.388	.631	.250	1.02	(c)
8	138	21.6	27.2	.407	.449	.224	1.23	(a)

Note: (a) Shallow upper cross beams, fixed ends.

(b) Deep upper cross beam and transverse bracing.

(c) No upper bracing.

An examination of these data shows that k generally increases with an increase in span. This is explained by the greater number of frames participating in the resistance to turning. Also, where spans are approximately equal, the k factor increases with an increase in rigidity of the upper bracing.

As implied above, k can be determined by measured deflections of a bridge. However, it is possible to predict k from the analysis of Section II, that is, through the use of the calculated value X_a. It was shown in Section II:

$$A = \frac{Pa}{b} - X_a n \qquad \text{Equation (1)}$$

$$B = \frac{Pa'}{b} + X_a n \qquad \text{Equation (2)}$$

Combining equations (1), (2) and (19), we get:

$$\frac{Pa}{b} - X_a n = \frac{Pa}{a + a'k} \qquad (20)$$

From equation (20) we get:

$$k = \frac{Paa' + X_a ha}{Paa' - X_a ha'} \qquad (21)$$

and

$$X_a = \frac{(Paa')(k-1)}{h(a+a'k)} \qquad (22)$$

Thus from formula (21) we can calculate k by means of X_a which would be computed with the use of formula (16A); or we can find X_a by measurement of actual bridge deflections using formula (17).

Having accomplished the determination of k we then find the actual load on each main truss.

The determination of A and B makes it possible to find the actual relative displacement of the opposite ends of a floor beam and thus the tilt of the track, if this be desired.

SECTION IV

(a) Model Similitude, (b) Design of the Model

(a) It was recognized early in this study, that an attempt to simulate an actual bridge structure, in the design of the model, maintaining all the requisite similitude features would entail a number of complications. Reference to several published treatises on the subject, for example that of Mr. Groat[4] and Mr. Carlson,[5] indicated the full extent of the difficulties involved.

In the latter reference, for example, there is presented a general treatment of model similitude which depends on the following two rules:

(A) The model shall be geometrically similar to its prototype, except as to dimensions which do not affect the behavior of the model.

(B) The force scale-reduction factor shall be the same for forces arising from each of the various influences.

In these two rules, (A) fixes all dimensions and (B) fixes the properties of the material and interprets the model results in terms of the prototype.

(4) Transactions A.S.C.E., Vol.96, 1932
"Theory of Similarity and Models,"
Benjamin F. Groat.

(5) Engineering News-Record, August 23,1934
"Similitude Requirements in Model Design,"
Roy W. Carlson.

To further illustrate the problem of similitude, pertinent excerpts of Mr. Carlson's treatment are continued.

Nomenclature

w and w_m are densities of material which are not necessarily uniform in each structure, but which vary in the same relative manner in model as in prototype.

L and L_m are linear dimensions of the whole or of corresponding parts of the two structures. L/L_m is termed n.

F and F_m are homologous forces.

E and E_m are elastic moduli.

e and e_m are unit elastic strains.

The gravitational force ratio is expressed: $\frac{F}{F_m} = \frac{w L^3}{w_m L_m^3}$

The elastic force ratio can be expressed: $\frac{F}{F_m} = \frac{E e L^2}{E_m e_m L_m^2}$

Utilizing these two factors (others which do not apply to a statically loaded bridge are not here given), the mathematical expression of rule B becomes:

$$\frac{F}{F_m} = \frac{w L^3}{w_m L_m^3} = \frac{E e L^2}{E_m e_m L_m^2} \qquad (23)$$

These relationships allow considerable latitude in the selection of scale-reduction factors. But rule A, by demanding geometrical similarity under all degrees of load, requires that the elastic strains be equal for model and prototype. Thus cancelling e and

e_m from (23) above, the relationships now become:

$$\frac{F}{F_m} = \frac{w L^3}{w_m L_m^3} = \frac{E L^2}{E_m L_m^2} \qquad (24)$$

For purposes of further illustration, there will now be assumed the following values for the terms in the above equations: (based on a celluloid model, where $L/L = n = 50$)

$w = 490$ lbs/ft^3	$w_m = 83$ lbs/ft^3
$L =$ Lineal dimens.	$L_m = L/50 =$ Lineal dimens.of model
$F =$ External load	$F_m =$ Load on model
$E = 30{,}000{,}000$ p.s.i.	$E_m = 263{,}000$ p.s.i.

Then:
$$\frac{F}{F_m} = \frac{490}{83} \cdot \frac{L^3}{\frac{L^3}{50^3}} \neq \frac{30 \times 10^6}{263{,}000} \cdot \frac{L^2}{\frac{L^2}{50^2}} \qquad (25)$$

$$\frac{F}{F_m} = \frac{490 \times 125{,}000}{83} \neq \frac{30 \times 10^6 \times 2500}{263{,}000}$$

$$\frac{F}{F_m} = 737{,}000 \neq 285{,}000 \qquad (26)$$

It is apparent, from the above, that the similitude requirements are not satisfied. In order to accomplish this purpose, it would be necessary to find a material with an E_m and w_m which would make the above equations compatible. Or as an alternative, the factors e and e_m could be put back in the third term of equation (25) and a correction factor applied to the model results which would satisfy equation (26). The correct model load for an equivalent prototype load would then be established.

Unfortunately, however, there still remains another difficulty. This lies in the fact that, although the factor $n = 50$ could be readily applied to the major bridge dimensions, it would be a practical impossibility to apply this reduction to the actual member sizes.

Should it be considered a prime necessity, it is believed that this last difficulty could be surmounted. However, consideration of the subject problem, namely, the study of torsional rigidity, lead to the decision that exact similitude, in all respects, to an actual bridge was not necessary for the stated objectives.

That this decision is tenable can be seen by reference to Section VI wherein it will be noted that the torsional rigidity factor is an abstract number, independent of externally applied loads and dependent principally on the deflection behavior of the structure. From the foregoing, it is concluded that if the deflection characteristics of the model can be demonstrated to comply with a theoretical analysis of the model, insofar as the usual features are concerned (particularly, for example, influence lines for joint deflections) it may be safely assumed that the torsional deflection of the model is in accordance with the potential torsional deflection of an actual bridge. The agreement of theoretical with actual deflection of the model having been established, as will be shown in the next section, it is considered that the torsional test data may be justifiably related to the theoretical development of the torsional rigidity factor.

(b) For the purpose of general size determinations, the model was designed to about 1/50th of the height, width and span of the Pennsylvania R.R. bridge* over the C.R.R. of N.J. and Brills Crossing, at Newark, N.J. The similitude of the model to

* Drawing of bridge furnished through courtesy of the Bridge Engineer's Office of the P.R.R. in Philadelphia.

the actual bridge extends only to these dimensions, to the character of the span supports, to the type of wind bracing, and the type of bridge (sub-divided Warren truss, two track). The results reported hereinafter are in no way to be construed as applying to the actual Pennsylvania R. R. bridge.

The design of the model, from this point on, was based on the contemplated loading for the torsion tests. The model was made of celluloid members, aluminum alloy gussets and connected with #4-48 steel machine screws and nuts. Wherever it was necessary to fabricate members or install spacers, acetone was used as a very effective solvent and cement. The ends of the bridge were supported in the customary manner, one end on rollers and one end hinged. Complete details can be seen on the photographs, Plates 1 and 6, and the drawing, Plate 2. (see middle spread).

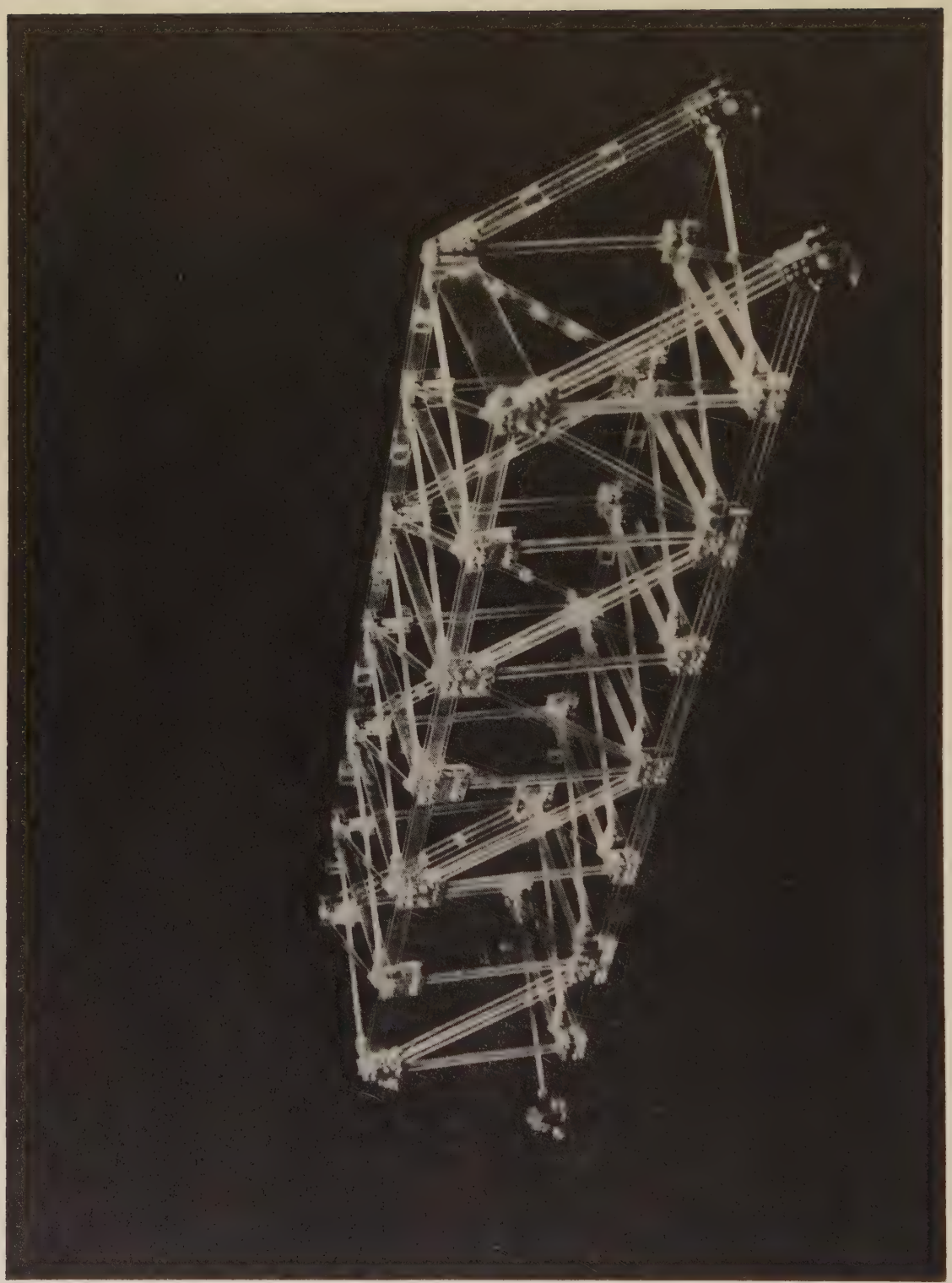
The celluloid used was of an ordinary commercial grade and its physical properties were determined by pulling several tension specimens and recording stress-strain curves. The average properties were:

Modulus of elasticity	-	263,200 p.s.i.
Yield point stress	-	1,800 p.s.i.
Ultimate tensile stress	-	3,690 p.s.i.

Stress-strain curves and other test data are given on Plate 3.

Care was taken in the tests to keep all developed stresses well under the yield point, in most cases, to a maximum of 1,000 p.s.i., or 800 p.s.i. under the yielding stress.

Because of the limited time available, it was not possible to determine accurately, temperature effects on



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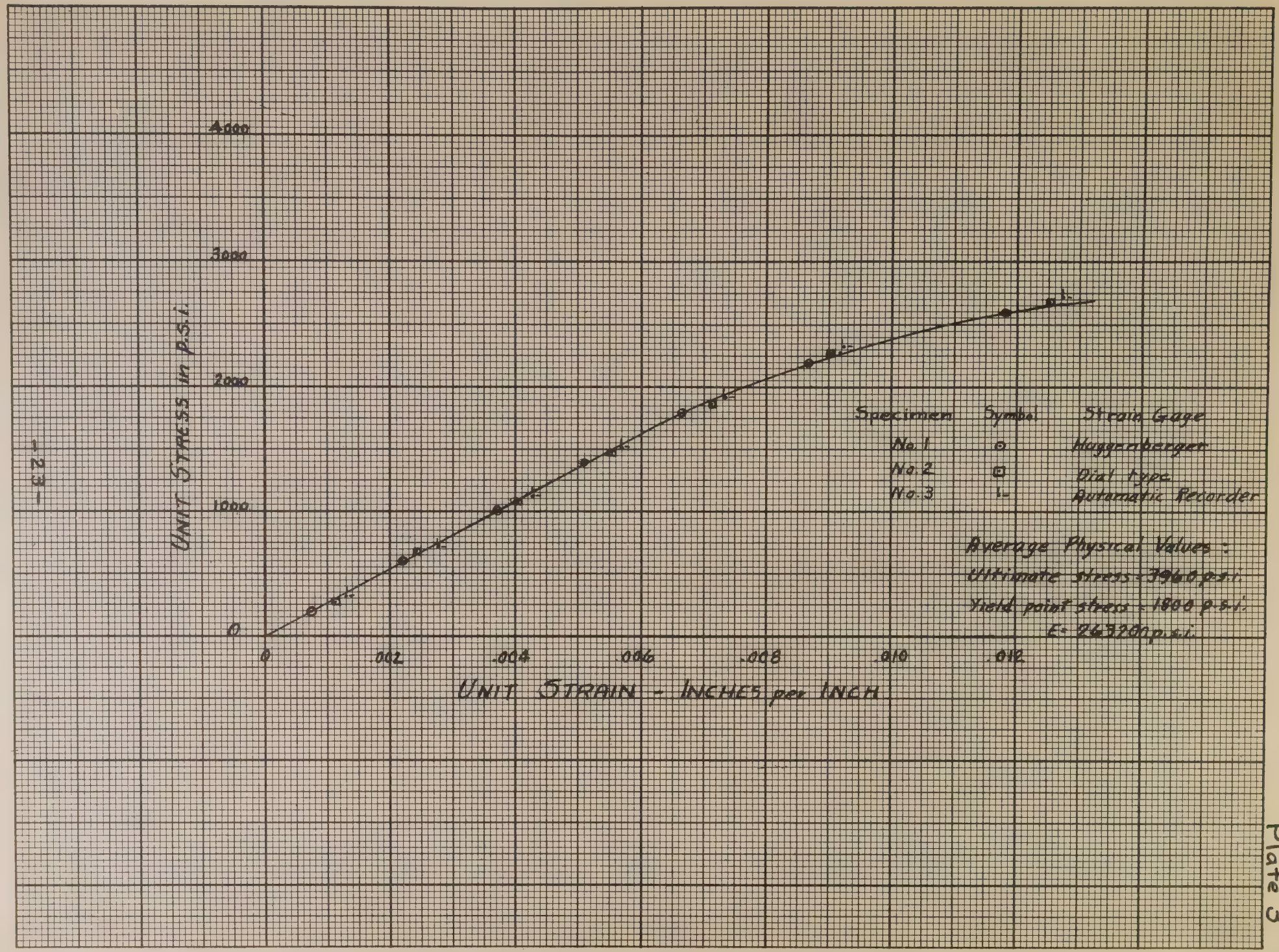

Plate 3

celluloid. Since, however, it was considered to be a possible source of error (a noticeable change in ultimate breaking loads was observed when the tensile test specimens were, in one case taken directly from a cold steel plate covered work bench and pulled; and, in another case, were pulled after being handled for several minutes), an attempt was made to perform all tests under approximately constant temperature conditions, and from a recording thermometer chart it was noted that all tests were performed within a temperature range of 7° F. (66°F. to 73°F.).

SECTION V

Experimental Verification of Deflection Characteristics of the Model

As stated in Section IV, it was desired to determine, initially, whether the model would deflect in accordance with the customary theoretical calculations. For this reason, preliminary experiments and calculations were made for the influence lines of the vertical deflections of both single truss and assembled bridge joints.

The theoretical influence lines for deflection were computed, using the usual application of Mohr's work theorem:

$$\delta_m = \sum S_a \frac{S \ell}{AE}$$

where δ_m = the deflection of the point in question

S_a = the total stress in a given member due to a load of unity applied in the direction in which the displacement is required at the point in question

S = the total stress in a member due to all externally applied loads

ℓ = length of member

E = modulus of elasticity

A = area of member

The results of these computations are plotted and shown on

Plate 4 for joints 1, 2, 3 and 4. (Fig. 9).

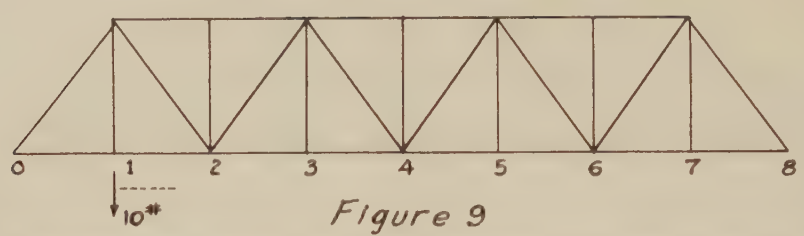

Figure 9

The curves shown on Plate 4, in broken lines, are the computed influence lines for a single truss. The full lines are the curves found experimentally by actually measuring the vertical deflections at the respective joints, using a 1/10,000th inch gauge, for a load of 10 pounds placed successively at joints 1 to 7 inclusive, on an isolated truss. The reasonably good agreement was considered as one indication of the theoretically correct behavior of the truss.

As a further check, the computed influence line data are compared to some of the later test results.

Taking data for a 20 pound load placed at the center of the floor beam of the assembled bridge at panel points 4, 5, 6 and 7 respectively and comparing the joint deflections with the computed influence line for panel point 4, the following agreement is observed:

20 lbs. at Panel Point	Computed deflection at Panel Point 4 in inches	Measured Panel Point deflection in inches
4	0.0220	0.0230
5	0.0182	0.0188
6	0.0125	0.0125
7	0.0065	0.0070

Plate 4

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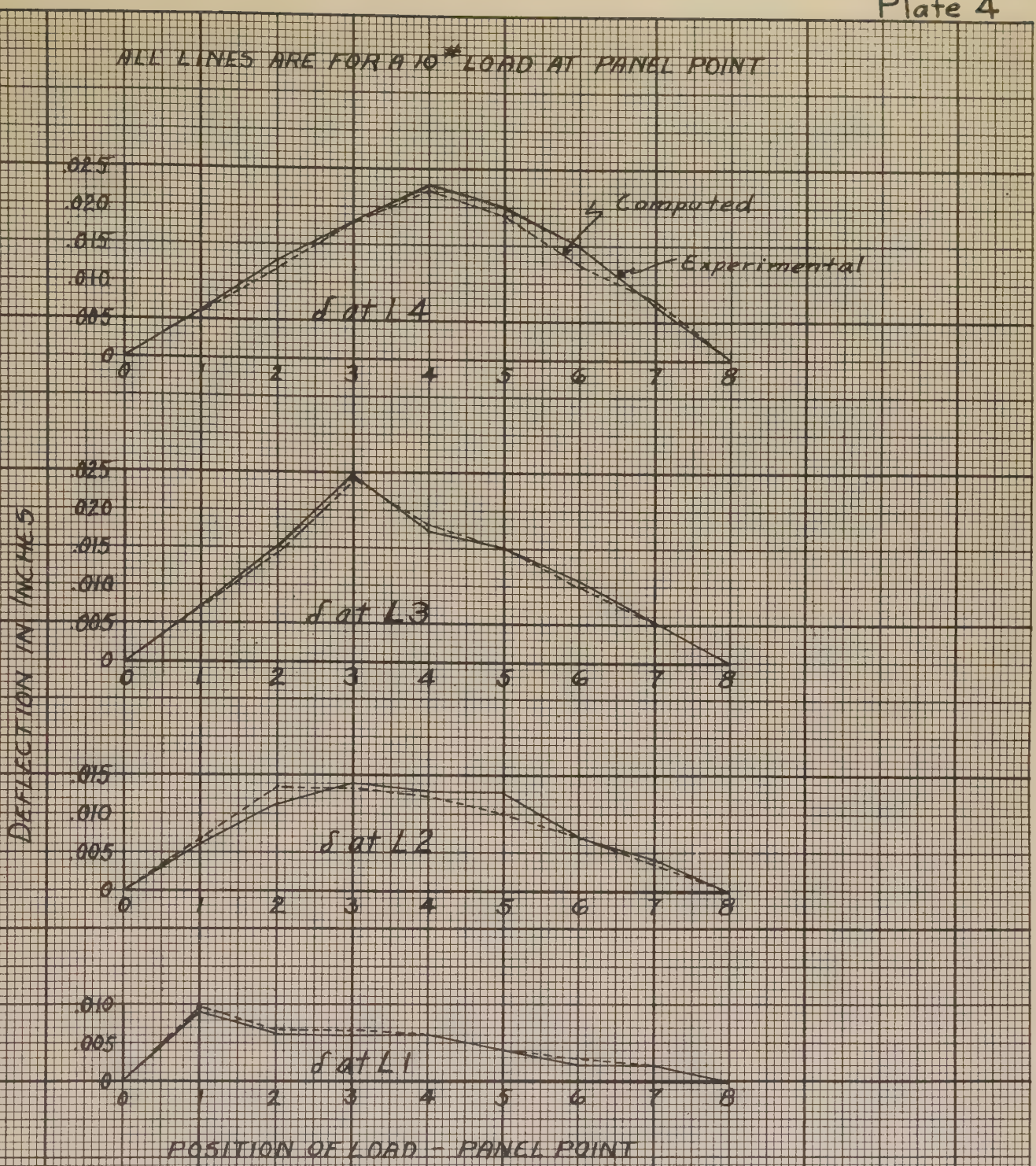


DEFLECTION

INFLUENCE LINES

MAIN TRUSSES

Computed and Experimental

20 lbs. at Panel Point	Computed deflection at Panel Point 5 in inches	Measured Panel Point deflection in inches
1	0.0065	0.0070
2	0.0142	0.0142
3	0.0241	0.0243

10 lbs. at center of floor beam at each Panel Point of the assembled bridge	Computed deflection at Panel Point 4 in inches	Measured deflection at Panel Point 4 in inches
	0.0949	0.0993

In another test, a 10 pound load was placed directly at panel point 4 of one truss, on the assembled model, and the measured deflection at panel point 4 was 0.0225 inches. The computed deflection, from the influence line is 0.0220 inches.

As a final check, it is pointed out that the deflection of the model conforms with Maxwell's law of reciprocal displacements which may be stated thus:

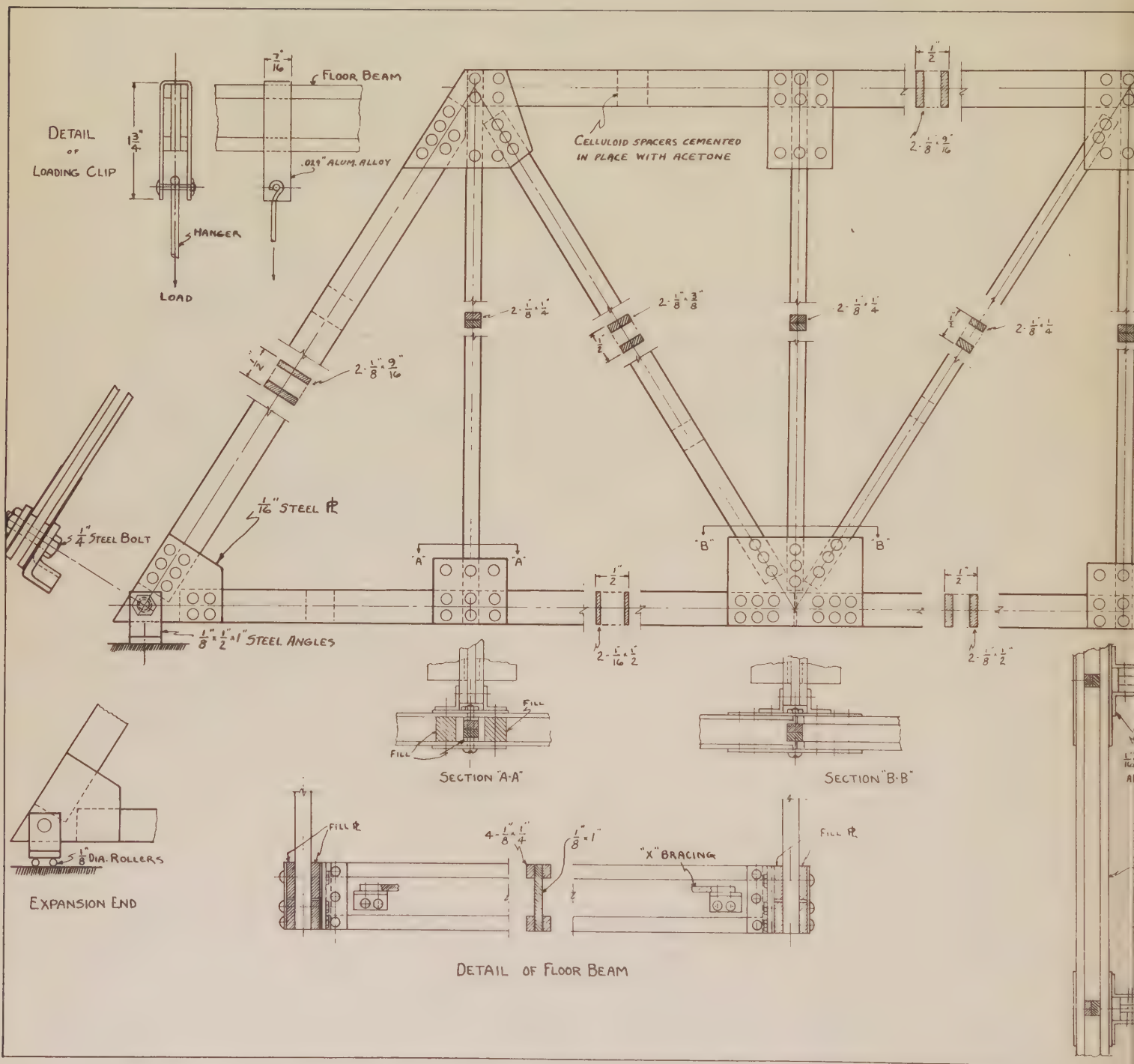
> The displacement δ_{mn} of a point m in a direction mm', produced by a unit load acting at n in another direction nn', is equal to the displacement δ_{nm} of a point n in the direction nn', produced by a unit load acting at m in the direction mm'.

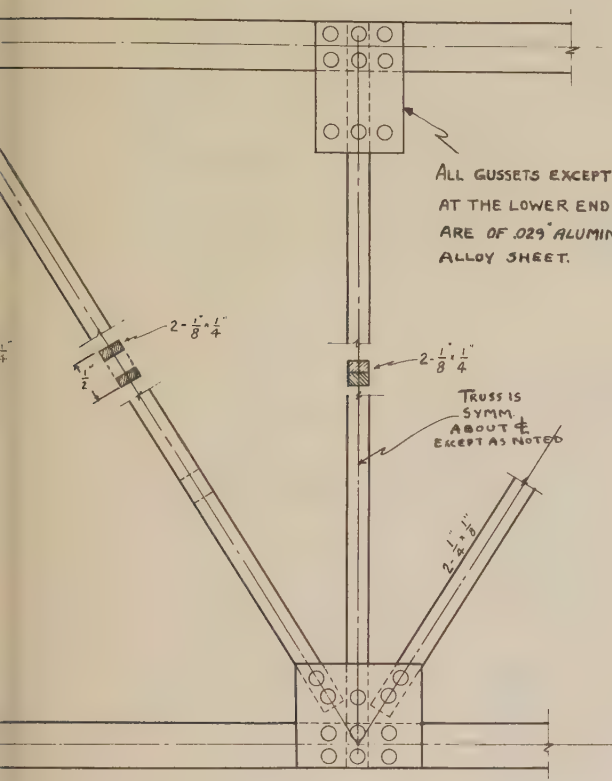
Referring to Plate 4, several instances of the agreement with this law will be seen (using the full line curves):

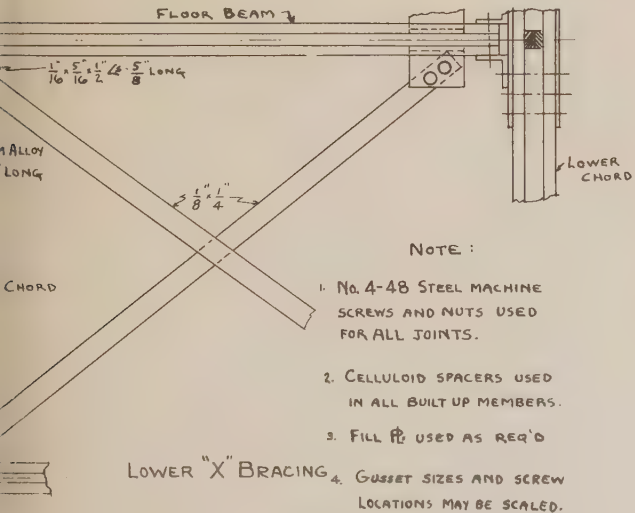
10 lbs. at Panel Point	Deflection in inches
1	0.0060 at pp 4
4	0.0060 at pp 1
2	0.0130 at pp 4
4	0.0129 at pp 2
3	0.0180 at pp 4
4	0.0172 at pp 3
1	0.0660 at pp 3
3	0.0610 at pp 1

etc.

Throughout all the tests in this investigation the model appeared to act in a truly elastic manner, that is, it always returned to its original position on removal of the load. No excessive buckling, twisting or deflection was observed at any time and the characteristics of the model appeared entirely suitable for the intended purposes.


DETAIL OF LOADING CLIP
7/16"
FLOOR BEAM
1 3/4"
.029" ALUM. ALLOY
HANGER
LOAD
CELLULOID SPACERS CEMENTED IN PLACE WITH ACETONE
2 - 1/8" x 9/16"
2 - 1/8" x 1/4"
2 - 1/8" x 3/8"
1/2"
1 IN
2 - 1/8" x 9/16"
1/16" STEEL PL
1/4" STEEL BOLT
1/8" x 1/2" x 1" STEEL ANGLES
"A"
"B"
2 - 1/16" x 1/2"
FILL
SECTION "A-A"
SECTION "B-B"
1/8" DIA. ROLLERS
EXPANSION END
FILL PL
4 - 1/8" x 1/4"
1/8" x 1"
"X" BRACING
DETAIL OF FLOOR BEAM

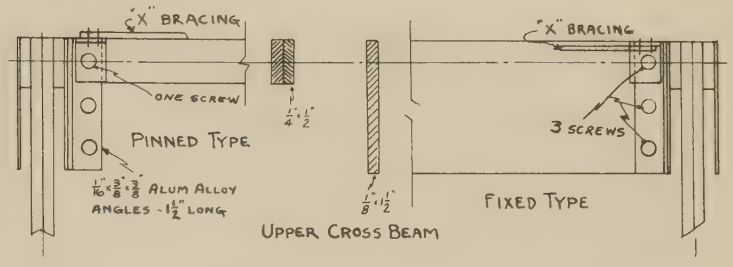

ALL GUSSETS EXCEPT THOSE AT THE LOWER END JOINTS ARE OF .029" ALUMINUM ALLOY SHEET.
2-1/8" x 1/4"
2-1/8" x 1/4"
TRUSS IS SYMM. ABOUT ℄ EXCEPT AS NOTED
2-1/4" x 1/8"

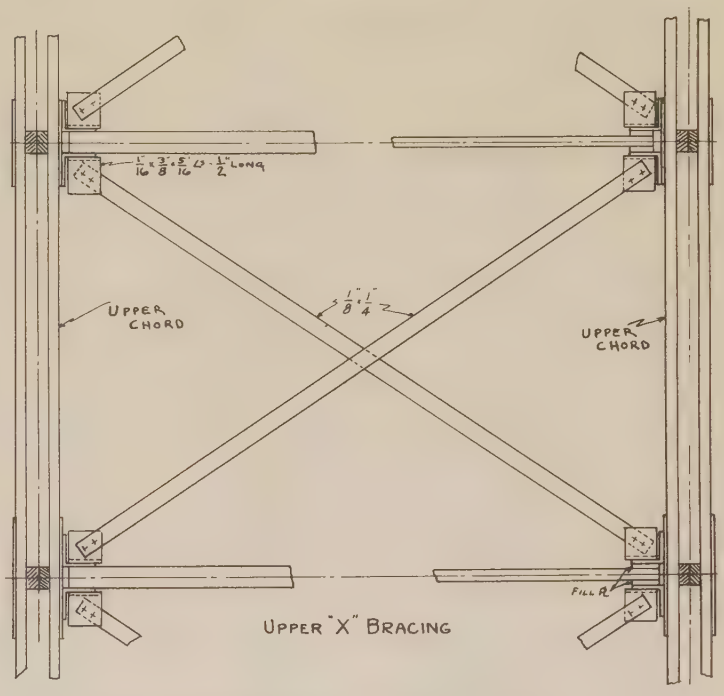


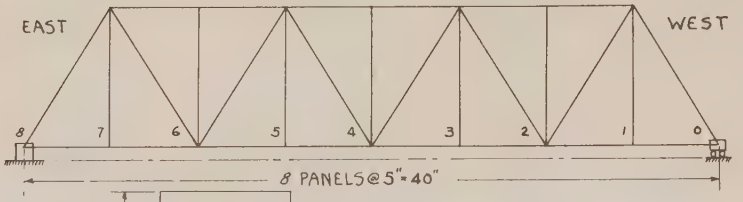
LOWER "X" BRACING

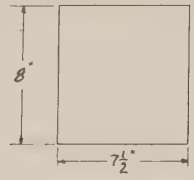
NOTE:

1. No. 4-48 Steel machine screws and nuts used for all joints.
2. Celluloid spacers used in all built up members.
3. Fill ℞ used as req'd
4. Gusset sizes and screw locations may be scaled.









DESIGN OF THE CELLULOID MODEL

SCALE ONE INCH

SECTION VI

Application of the Rigidity Theory to the Model

In this section the theoretical analyses derived in Sections II and III will be applied to the model. There is given below a tabular summary of the properties of the various model bridge members. It is to be noted that the trusses are asymmetrical about their center lines.

Member	Area $in.^2$	Length in.	Mom.of I $in.^4$	L/A
$L_0 L_1$	0.0996	5.0	0.00234	50.2
$L_1 L_2$	0.0996	5.0	0.00234	50.2
$L_2 L_3$	0.1290	5.0	0.00285	38.75
$L_3 L_4$	0.1290	5.0	0.00285	38.75
$U_1 U_2$	0.1480	5.0	0.00435	33.8
$U_2 U_3$	0.1480	5.0	0.00435	33.8
$U_3 U_4$	0.1480	5.0	0.00435	33.8
$L_0 U_1$	0.1480	9.44	0.00435	63.8
$U_1 L_2$	0.1050	9.44	0.00485	89.5
$L_2 U_3$	0.0790	9.44	0.00268	119.5
$U_3 L_4$	0.0790	9.44	0.00268	119.5
$L_1 U_1$	0.0740	8.0	0.00387	108.0
$L_2 U_2$	0.0700	8.0	0.000366	114.0
$L_3 U_3$	0.0660	8.0	0.000346	120.0
$L_4 U_4$	0.0700	8.0	0.000366	114.0
Floor Beams	0.2380	7.5	0.0358	-
Wind Bracing Diagonals	0.0352	9.0	-	256.0

For the application of formula (16A) it is necessary to determine the following factors:

s - length of diagonals, 9.44"

h - height of bridge, 8"

b - width of bridge, 7-1/2"

$\frac{h}{b} = n = 1.066$

I_2 - average moment of inertia of the verticals of the bridge, 0.000366 $in.^4$

I_5 - average moment of inertia of floor beams, 0.0358 $in.^4$

I_9 - average moment of inertia of diagonals, 0.00340 $in.^4$

m - number of effective frames, 7

c_1 , c_3 , c_4 - the elastic constants of the upper horizontal, the lower horizontal, and the main trusses respectively.

By definition, the elastic constant of a truss is the load required at the center panel point to produce unit displacement. From Plate 5, it is found that a load of 10 lbs. at p.p. 4 causes a deflection of 0.023 inches. Thus c_4 becomes

$$1 \div 1/10 \cdot 0.023 = 435 \text{ lbs./in.}$$

The upper and lower horizontal trusses were considered as double Warren trusses, in which only one system of diagonals acts when a load is applied at the center (See Fig. 10).

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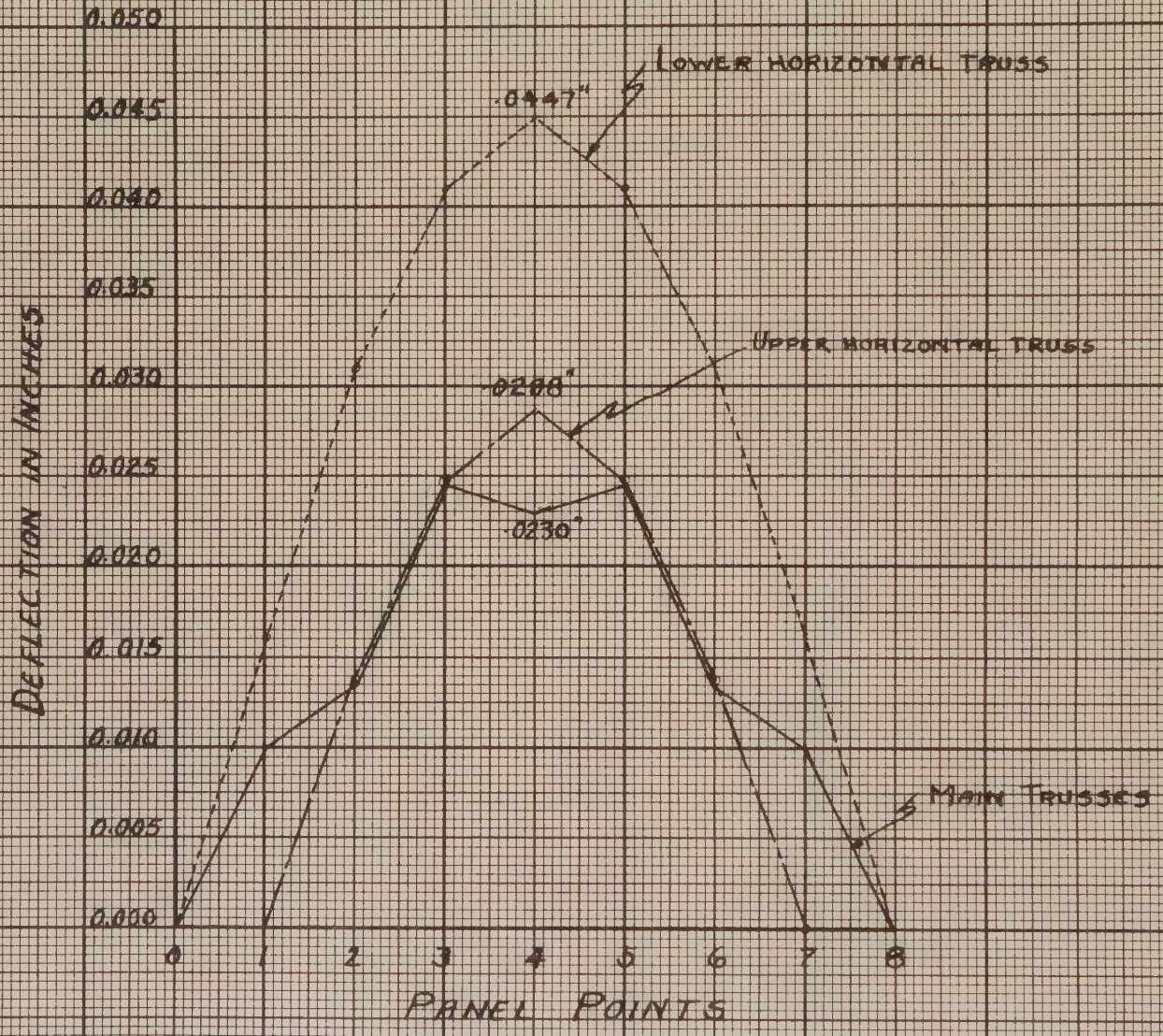
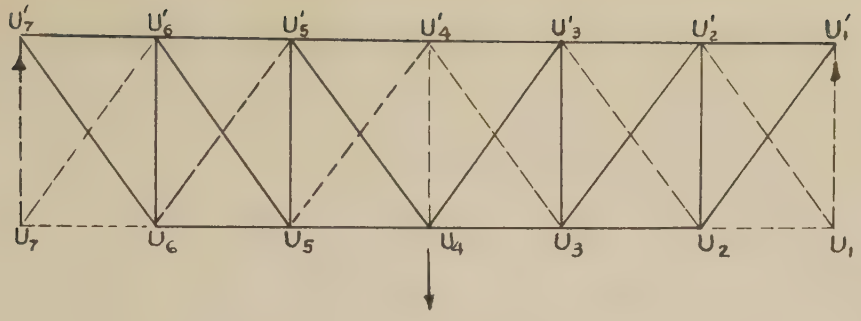
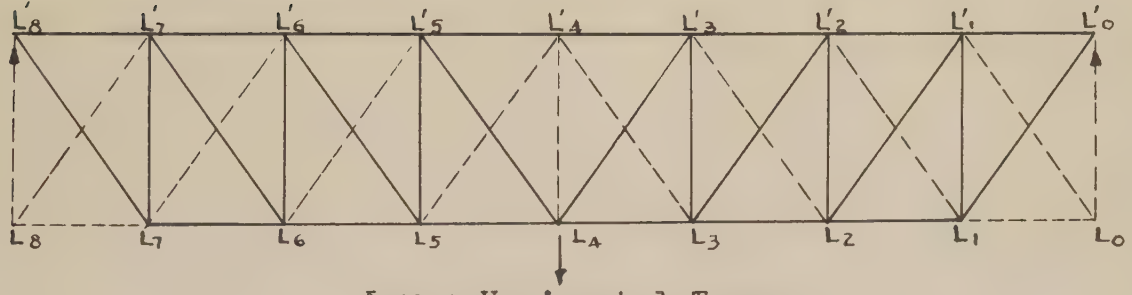


CURVES SHOWING DEFLECTION
AT A PANEL POINT DUE TO 10#
AT THE PANEL POINT

MAIN AND BOTH HORIZONTAL TRUSSES



Upper Horizontal Truss



Lower Horizontal Truss

Figure 10

(members shown dotted are inactive)

The reason for assuming this action of the trusses is apparent when we consider that the diagonals having extremely small l/r values can not act in compression. Loads at a point other than the center cause these trusses to act as indicated in Fig. 11.

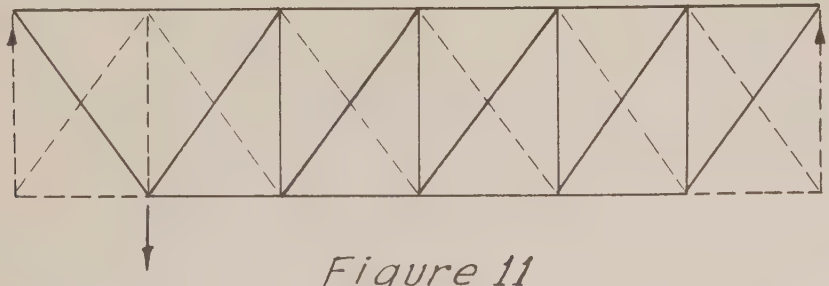


Figure 11

Applying Mohr's work theorem as outlined in Section V we determine the deflection at point 4 due to a load of unity at 4 (See Plate 5).

Deflection for upper truss = 0.00288 in.

Deflection for lower truss = 0.00447 in.

Thus, $c_1 = \frac{1}{0.00288} = 347$ lbs/in.

$c_3 = \frac{1}{0.00447} = 224$ lbs/in.

Formula (16A) is repeated:

$$X_a = \frac{2Pen}{b} \cdot \frac{1}{2n^2 + \frac{c_4}{c_1} + \frac{c_4}{c_3} + \frac{c_4 h^3}{6mEI_2} + \frac{c_4 h^2 s^2 b I_5}{12mE(2bI_9 + sI_5)^2}}$$

Substitution of the constants for this bridge gives:

$$X_a = Pe\,(0.0047)$$

We can now apply this value of X_a to formula (21), Section III, and thereby predict the torsional rigidity factor k for the model bridge:

$$k = \frac{Paa' + X_a ha}{Paa' - X_a ha'} = \frac{Paa' + (7.5a)(.0047)Pe}{Paa' - (7.5a')(.0047)Pe}$$

$$= \frac{aa' + .0353ae}{aa' - .0353a'e} \qquad (27)$$

It is apparent from this application formula that the rigidity factor for this particular model is dependent only on the amount of eccentricity e of the load. For any given bridge, where e is fixed, the factor k is dependent only on X_a which is fixed by the bridge characteristics.

The loading tests on the model were made with e = 2.125 inches ("A" loading) and e = 1.375 inches ("B" loading). (See Section VII). Thus, by the substitution of proper a, a', and e values in formula (27) we get:

For "A" loading k = 1.06

For "B" loading k = 1.03

In the following sections, the agreement of the test results with these factors will be discussed.

SECTION VII

Model Tests and Results

The method of testing the model can be seen on Plate 6. It was a matter of extreme importance to insure perfect rigidity of the model span supports and deflection datum surfaces since displacements were measured by the use of dial gauges reading to 1/10,000th of an inch on the scale.

Two 6" x 6" wooden columns firmly anchored to the floor and braced by diagonal struts were used as bridge piers. These two columns were capped by 1/2" x 10" x 10" steel plates to serve as bearing surfaces.

The deflection reference planes were hard wood frames circumscribing the bridge frames to be measured, and supported by a well braced steel angle framework, independently of the bridge, piers or any other part of the setup. The surfaces of the wooden frames were carefully made true, square and smooth for reasons which are explained below.

The method of measuring deflections can be seen in Fig. 12.

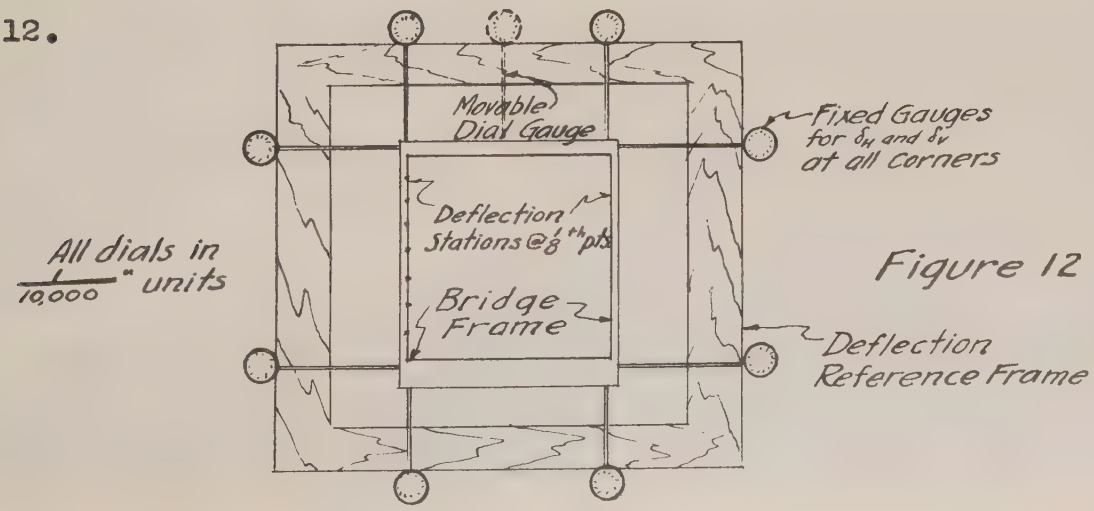
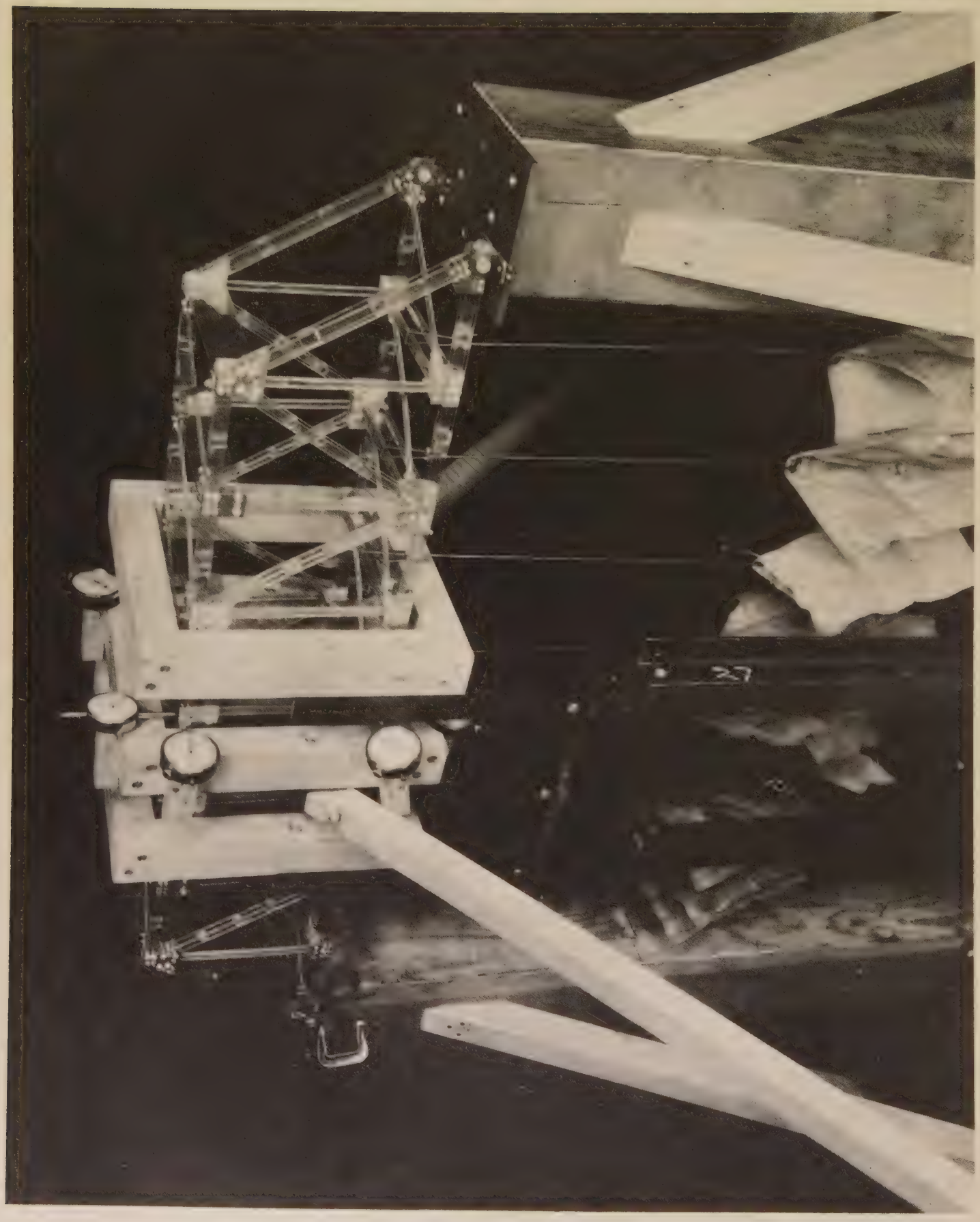

Movable Dial Gauge
Fixed Gauges for δ_H and δ_V at all Corners
Deflection Stations @ 1/8th pts
Bridge Frame
Deflection Reference Frame
All dials in 1/10,000" units

Figure 12

Plate 6

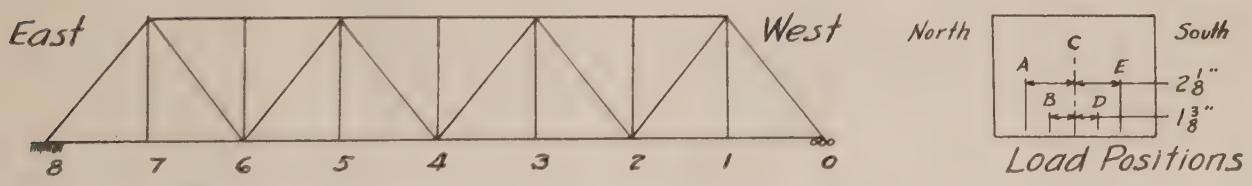


As will be seen in Fig. 12, the vertical and horizontal displacements of each joint were measured by fixed dial gauges. Displacements at the 1/8th points of all the members were measured by a dial gauge fastened to a smooth steel block which could be moved all around the reference frame to each deflection station not measured by a fixed dial. The trueness of the reference frame and steel block made it possible to attain an accurace substantially equal to that of the gauges.

A complete set of zero, or no load readings was taken for each test and, after application of the load, another set of readings was taken. Thus for each loading test, a total of 72 deflection gauge readings was recorded.

Loads were applied by means of clips placed over the floor beams, with hangers on the clips carrying accurately weighed lead shot bag loads (See detail Plate 2).

The tests were divided into four series and are summarized in Fig. 13.



Series a — Both upper and lower "X" Bracing
Top cross beams partly fixed (by friction)

Series b — Both upper and lower "X" Bracing
Top cross beams pinned

Series c — Lower "X" Bracing only
Top Cross beam pinned

Series d — Rigid fixed-end upper cross beams
Both upper and lower "X" Bracing

Figure 13

The loadings will be shown hereinafter. In all test series, the deflections of frame 4 were measured, and in series (b), the deflections of frame 5 were also measured as will be seen in the following table.

In the table below, the letters A, B, C, D and E refer to the location of the load on the floor beam, and the subscript refers to the frame loaded.

d_1 is vertical deflection of lower south joint

d_2 is vertical deflection of lower north joint

Series	Test No.	Deflections of Frame Measured	Load at	d_1(in)	d_2(in)	k
(a)	1	4	30#at A4	.0170	.0520	1.18
"	2	4	30#at B4	.0236	.0445	1.15
"	3	4	30#at C4	.0345	.0340	-
"	4	4	30#at D4	.0442	.0238	1.16
"	5	4	30#at E4	.0524	.0178	1.23
"	6	4	15#at A4 & E4	.0358	.0354	-
"	7	4	15#at A4	.0088	.0276	1.16
"	8	4	40#at A4	.0226	.0690	1.19
(b)	9	4	30#at A4	.0168	.0546	1.11
"	10	4	30#at B4	.0230	.0457	1.09
"	11	4	30#at C4	.0348	.0342	-
"	12	4	15#at A4 & E4	.0361	.0359	-
"	13	4	20#at C7	.0071	.0069	-
"	14	4	20#at C6	.0124	.0128	-
"	15	4	20#at C5	.0182	.0194	-
"	16	4	20#at A7	.0031	.0102	1.12
"	17	4	20#at A6	.0061	.0197	1.12
"	18	4	20#at A5	.0081	.0270	1.09
"	19	4	10#at C1 to 7 inc.	.0501	.0492	-
"	20	4	10#at A1 to 7 inc.	.0228	.0772	1.07
"	21	4	10#at C4,5,6,7	.0309	.0311	-
"	22	4	10#at A4,5,6,7	.0143	.0461	1.12
"	23	5	30#at A5	.0165	.0555	1.08
"	24	5	30#at B5	.0241	.0482	1.08
"	25	5	30#at C5	.0363	.0365	-
"	26	5	20#at C7	.0070	.0070	-
"	27	5	20#at C6	.0143	.0142	-
"	28	5	20#at A7	.0035	.0112	1.13
"	29	5	20#at A6	.0066	.0225	1.06
"	30	5	10#at C1 to 7 inc.	.0496	.0494	-
"	31	5	10#at A1 to 7 inc.	.0236	.0753	1.13
"	32	5	10#at C5,6,7	.0246	.0240	-
"	33	5	10#at A5,6,7	.0114	.0355	1.16

Series	Test No.	Deflections of Frame Measured	Load at	d_1(in)	d_2(in)	k
(c)	34	4	20#at A4	.0100	.0358	1.01
"	35	4	20#at B4	.0145	.0312	1.00
"	36	4	20#at C4	.0221	.0225	-
"	37	4	30#at A7	.0046	.0164	1.01
"	38	4	30#at A6	.0084	.0300	1.01
"	39	4	30#at A5	.0125	.0453	1.00
"	40	4	30#at A7	.0106	.0105	-
"	41	4	30#at C6	.0196	.0191	-
"	42	4	30#at C5	.0290	.0291	-
"	43	4	10#at C1 to 7 inc.	.0501	.0510	-
"	44	4	10#at A1 to 7 inc.	.0221	.0804	1.00
(d)	45	4	30#at A4	.0196	.0530	1.34
"	46	4	30#at B4	.0260	.0460	1.22
"	47	4	30#at C4	.0360	.0354	-
"	48	4	20#at C7	.0073	.0071	-
"	49	4	20#at C6	.0144	.0142	-
"	50	4	20#at C5	.0198	.0201	-
"	51	4	20#at A7	.0037	.0103	1.30
"	52	4	20#at A6	.0075	.0209	1.30
"	53	4	20#at A5	.0105	.0301	1.26
"	54	4	10#at C1 to 7 inc.	.0501	.0508	-
"	55	4	10#at B1 to 7 inc.	.0410	.0690	1.27
"	56	4	10#at A1 to 7 inc.	.0270	.0800	1.22

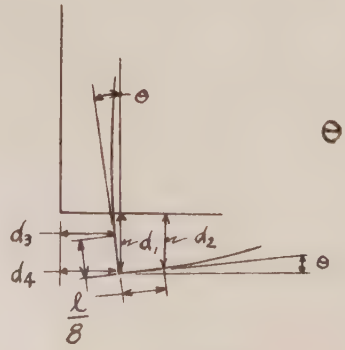
The above tables give all pertinent test data as well as the measured vertical deflection under each lower joint. The column at the extreme right lists the value of k computed from the deflections in each case as indicated in Section III. These values are discussed in the next Section.

It is considered a matter of great interest that the test results, when plotted as shown on Plates 7 to 21 inclusive, give a good picture of the elastic behavior of the frame members showing joint rotation, inflection points and deflection. It is interesting to note that test series (a) was an inadvertent one which came about because of an initial error in the construction of the model. The first test series was intended to be one for the condition of pin ended upper cross beams with "X" bracing in

the upper horizontal plane. However, when tests 1 to 8 inclusive were plotted, at the conclusion of the first 8 loadings, it was at once noted (Plates 7 and 8) that the elastic curve of the upper beam was not a straight line, a necessary condition for pin ends since no bending moment can be present in such members. Investigation disclosed the fact that sufficient friction, caused by the tightness of fit of the ends of the beams in the connection angles, existed, which partially fixed the beams. This condition having been remedied, the next series, (b), was performed, and the plotted elastic curves, Plates 9 to 18, show the change in behavior of the upper cross beams.

Although it is not intended to devote particular attention to the elastic curves of the frame members, since this does not come within the scope of this study, it is obvious that the data used in plotting the curves, Plates 7 to 21, can be used in determining bending moments by application of the slope deflection method. The first two horizontal displacement readings at the end of each vertical, and the first two vertical displacement readings at the end of each horizontal, provide the necessary data for determining the rotation at the joint and this, of course, can be used in the general slope deflection equations to determine the moments at the ends of fixed ended members. Thus, in Fig. 14 can be seen the method of determining the angular rotation of the joint.

Figure 14



$$\theta = \tan^{-1}\frac{8(d_1-d_2)}{\ell} = \tan^{-1}\frac{8(d_3-d_4)}{\ell}$$

$$M_{AB} = \frac{2EI}{\ell}\left(2\theta_A + \theta_B - 3\frac{d}{\ell}\right) - C_{AB}$$

In the above general slope deflection equation, E, I and ℓ are known, C_{AB} is the fixed end moment at A due to external loads on the beam which can be readily calculated from the external loads and e_A, e_B, and d (difference in elevation of the joints) can be obtained from the elastic curves. It must be pointed out, however, that in order to study more fully the moments as indicated above, in tests made specifically for this purpose, it would be necessary to measure the joint rotations very carefully by a more direct method than used in the above discussion. The values here concerned are very small and extreme accuracy is an essential requirement. It is further to be noted that the effective moment of inertia for the whole length of the member should be considered. For example, the vertical members have, at each end, rather long gussets and spacers which may participate in the behavior of the actual member itself, which is only 1/4" square in section.

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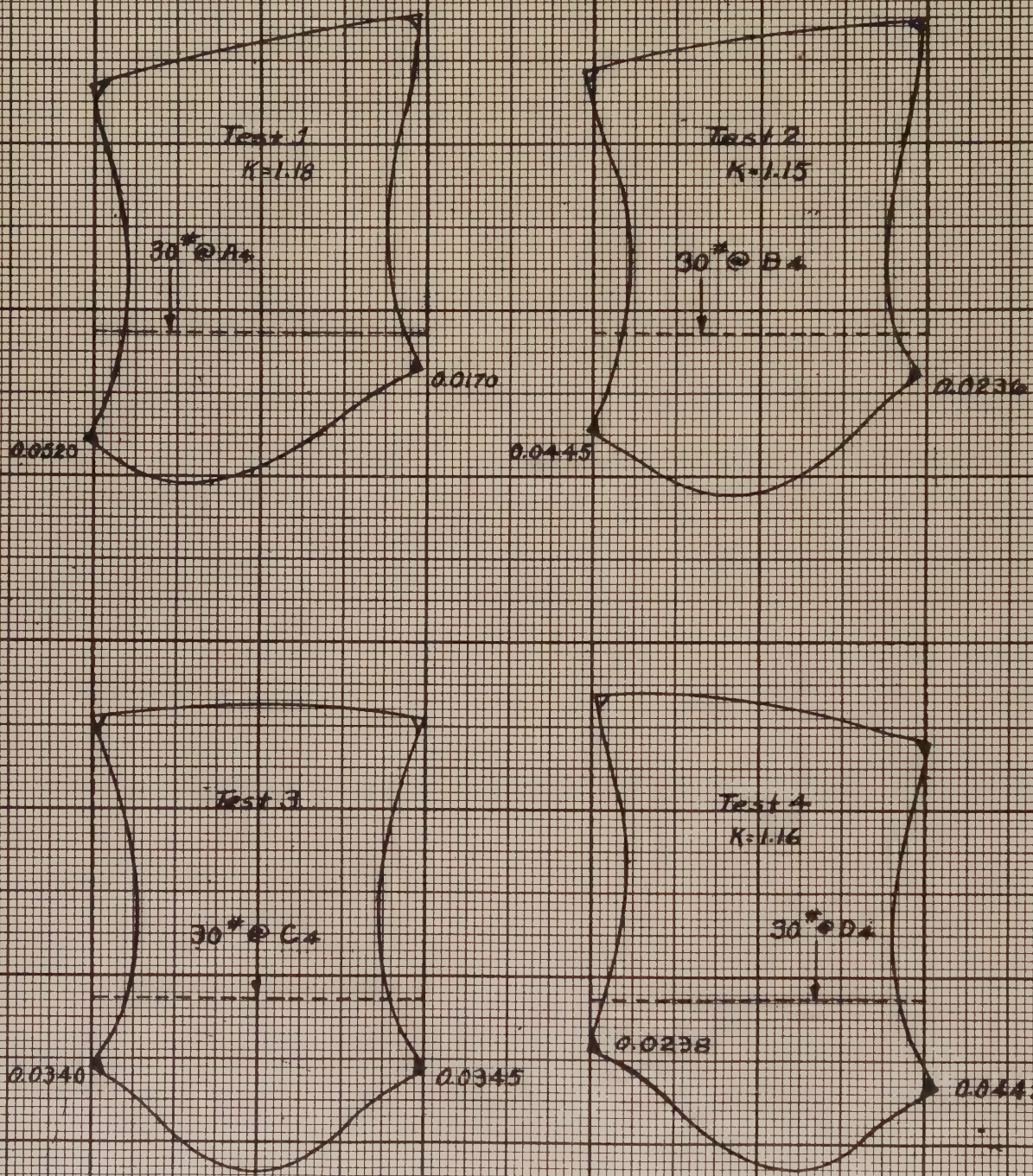
Series (a)



Deflections at Frame 4

Scale:
Outline $\frac{1}{4}$" = 1"
Deflections $\frac{1}{8}$" = 0.01"

These scales apply to the remaining elastic curves.

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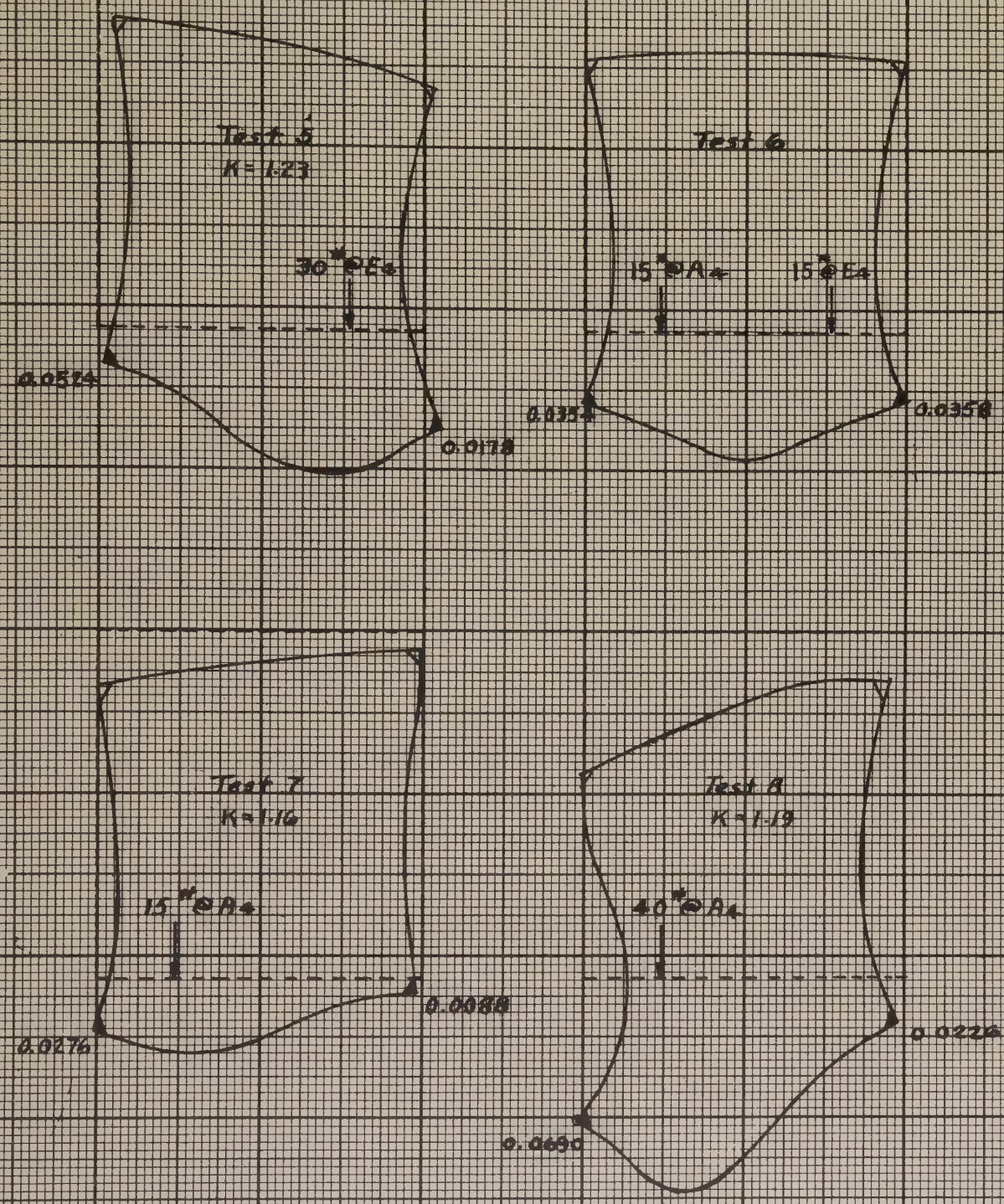
Series (a)



Deflections at Frame 4

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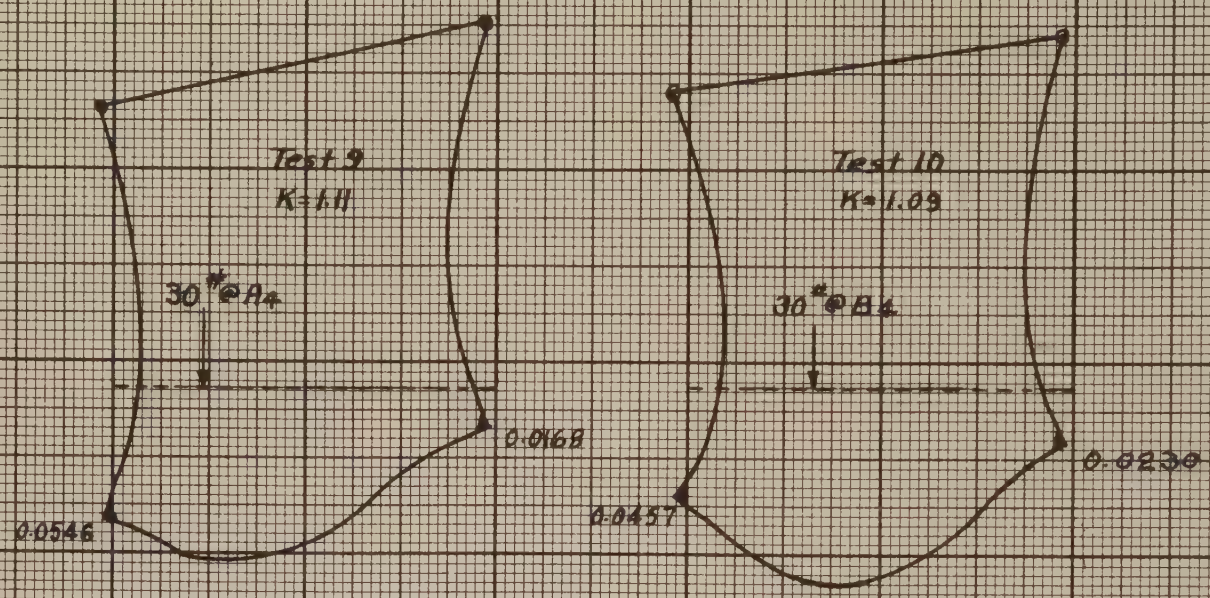
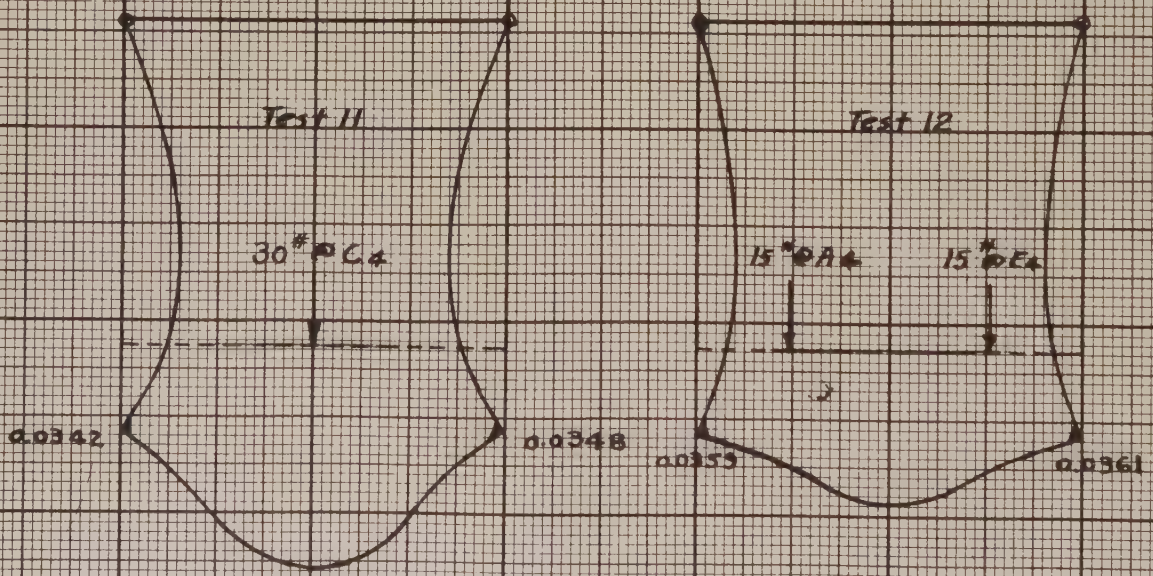
Series (b)





Deflections at Frame 4

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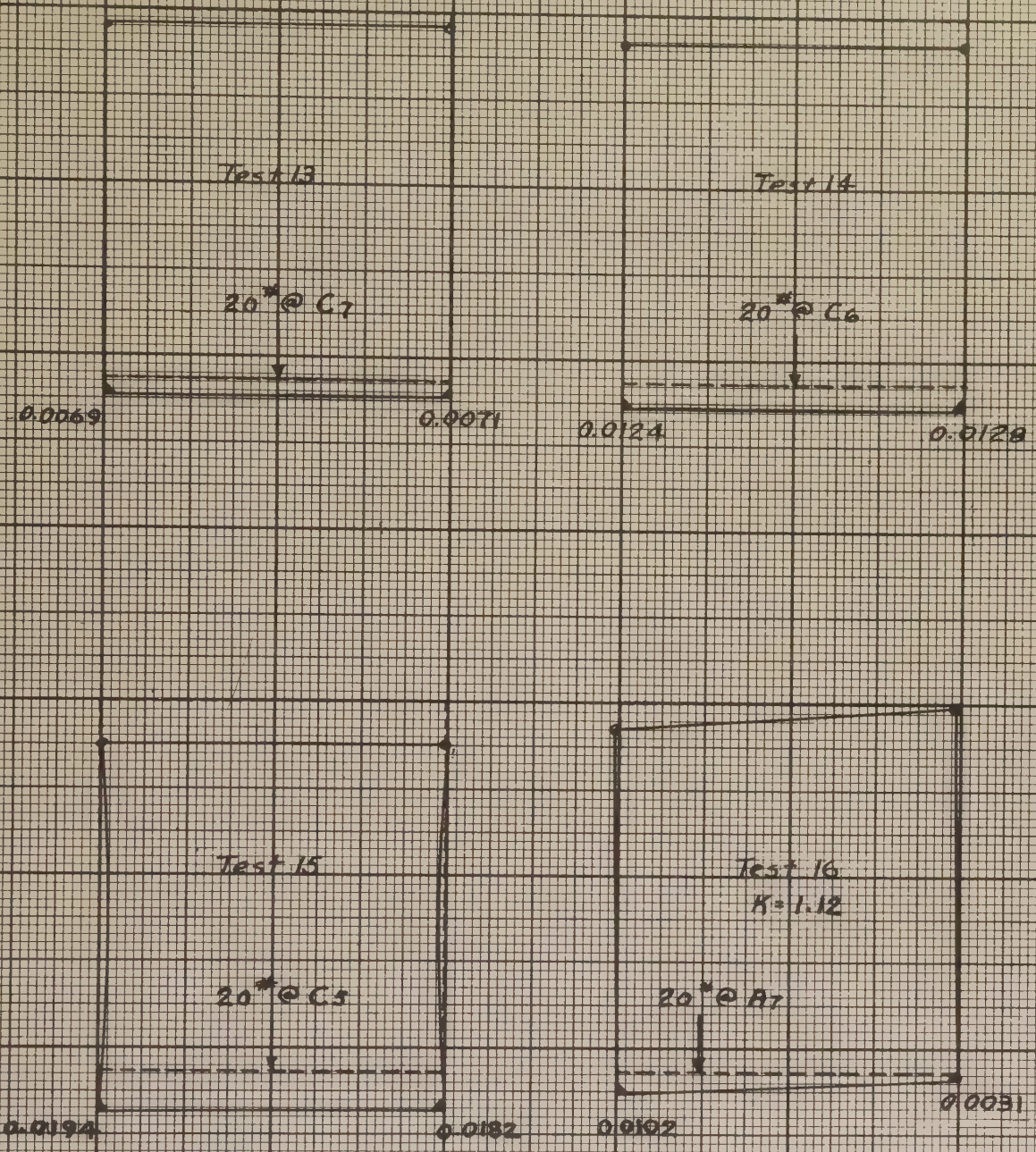
Series (b)



Deflection at Frame 4

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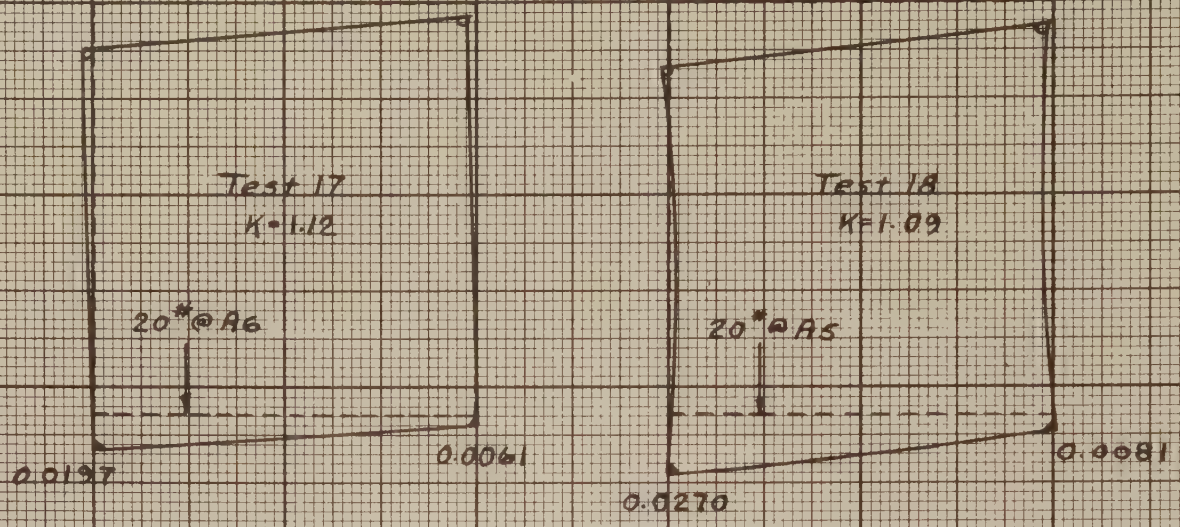
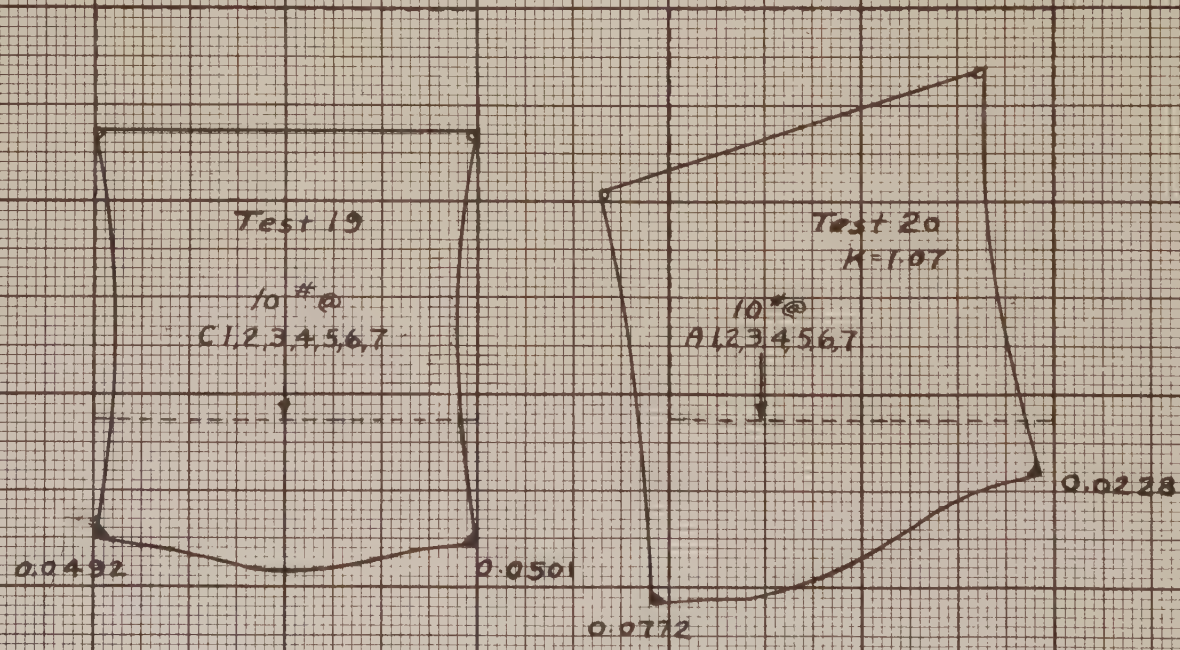
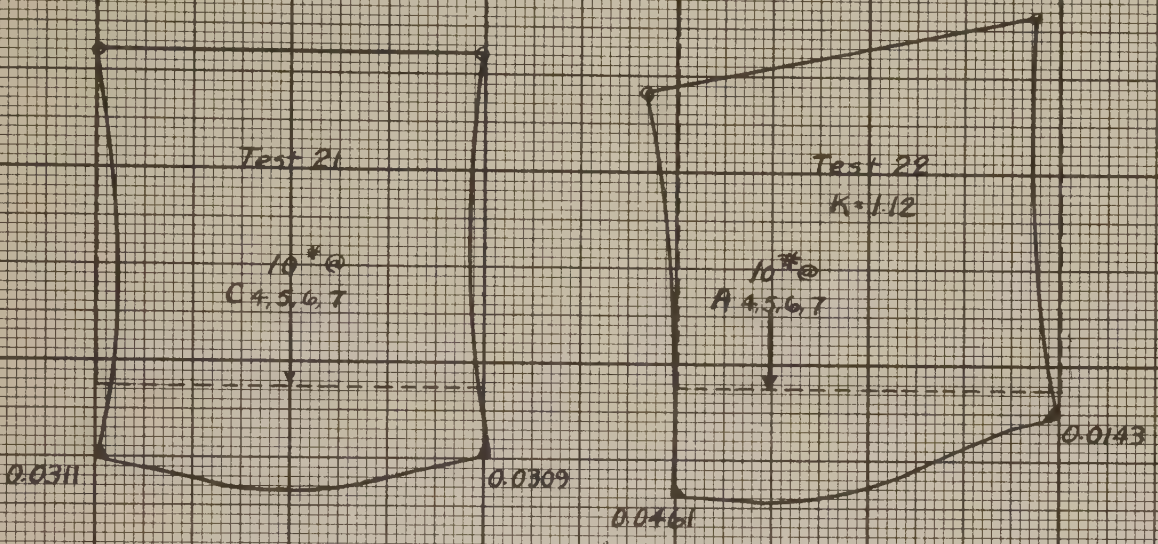
Series (b)





Deflection at Frame 4

Series (b)



Deflections at Frame 4

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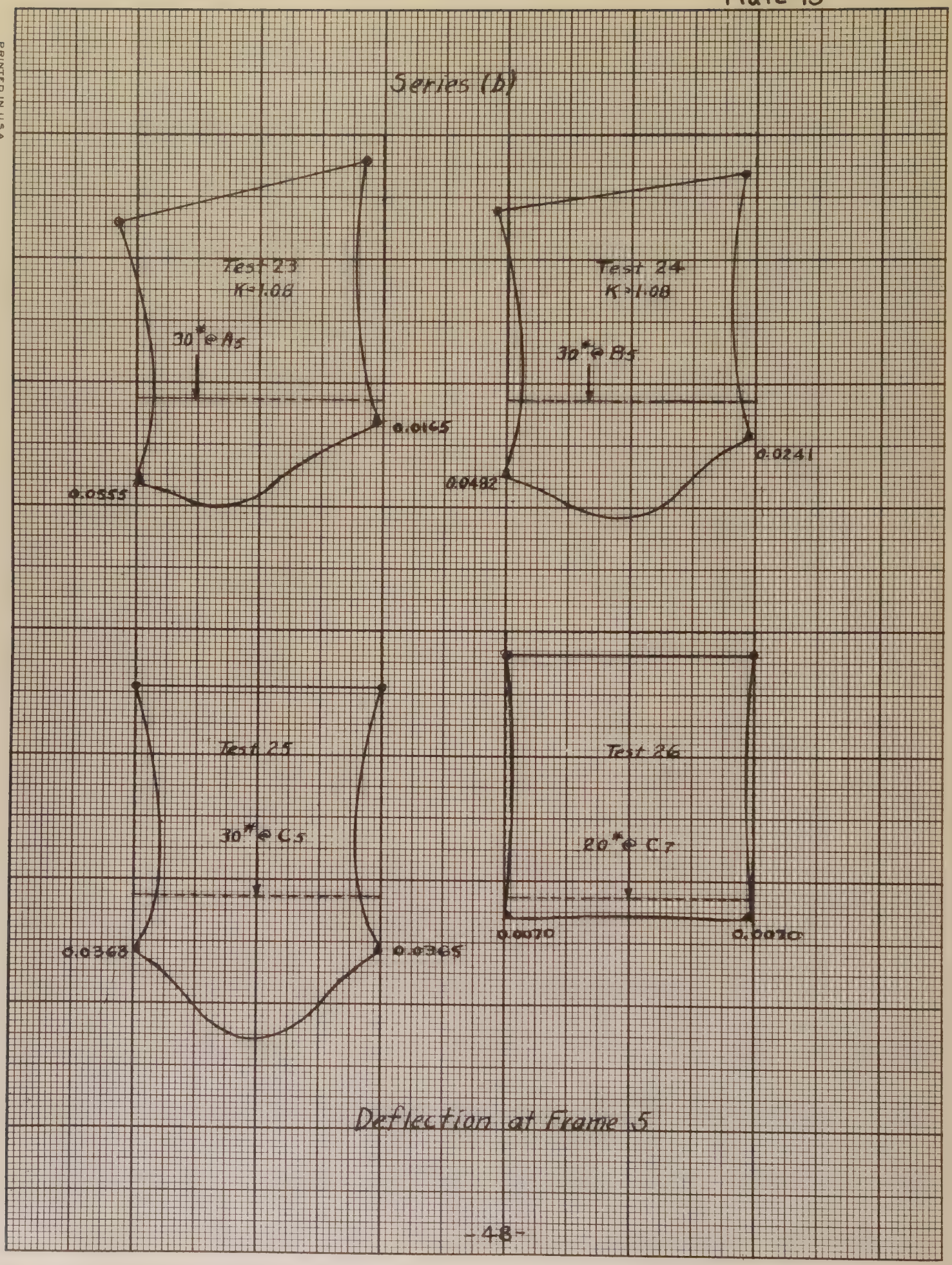
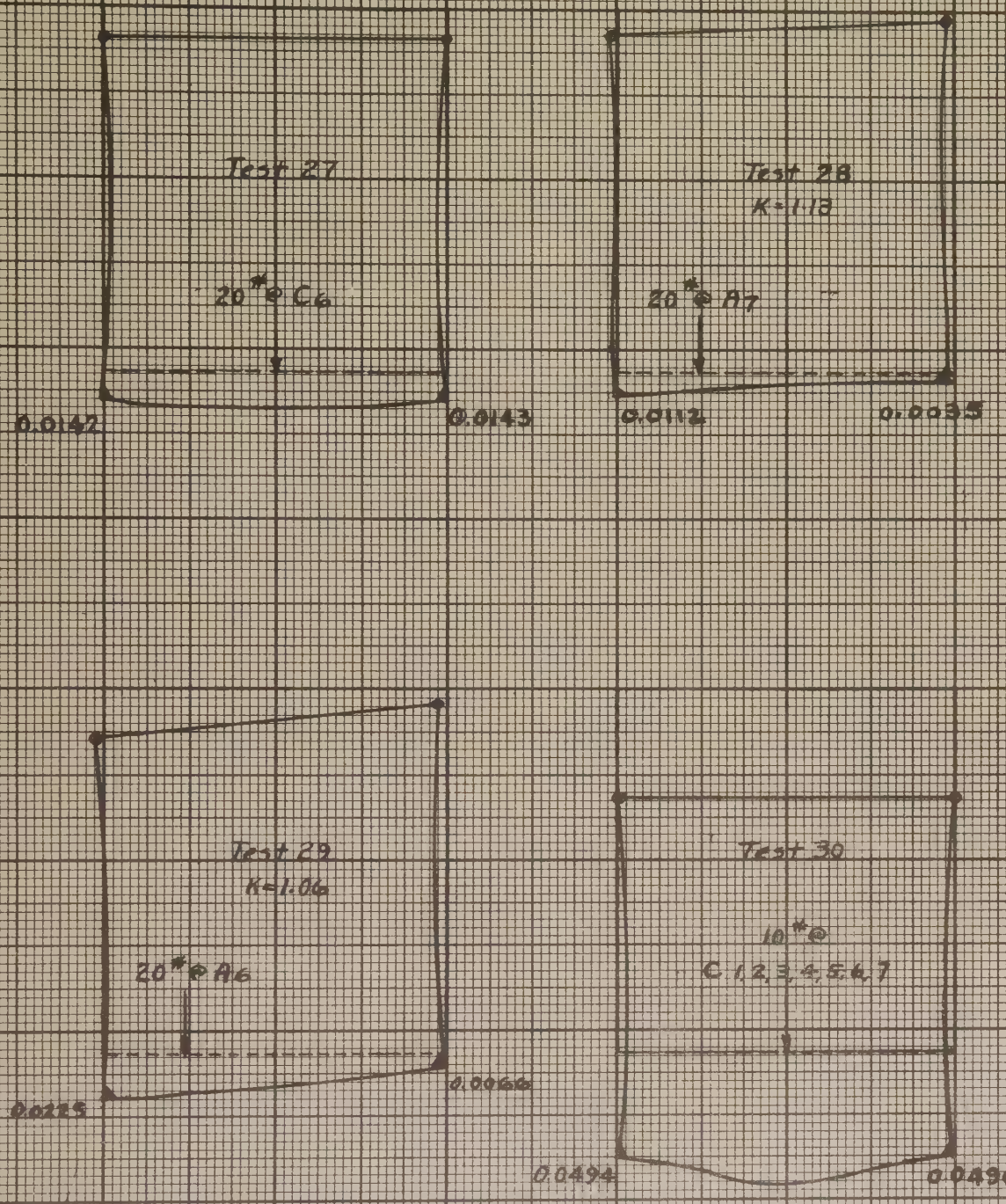


Deflection at Frame 5

Series (b)



Deflection at Frame 5

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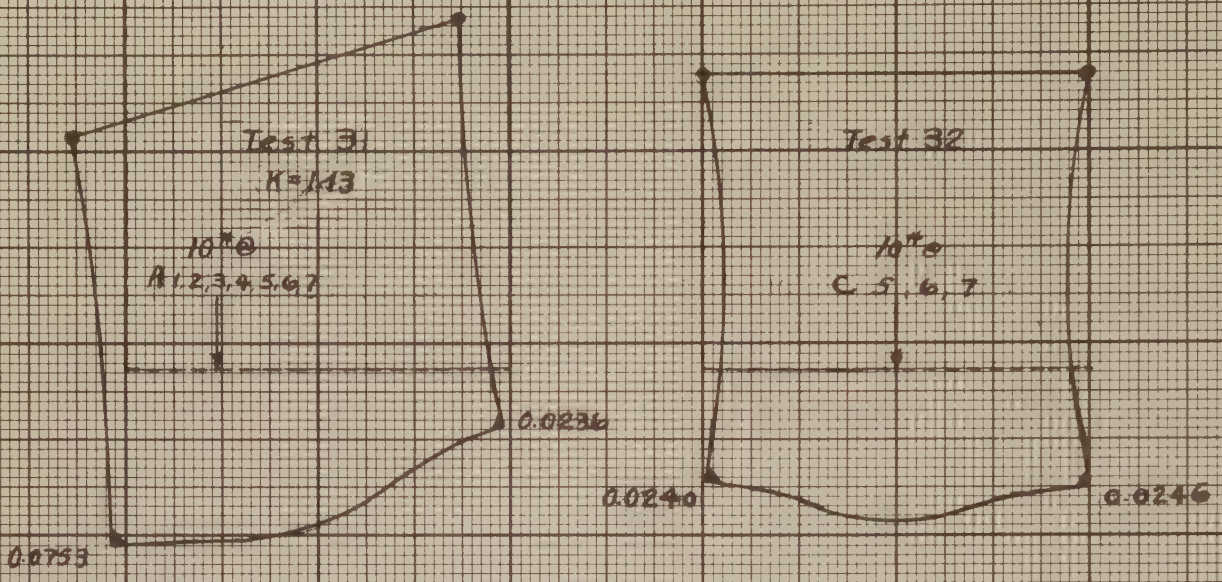
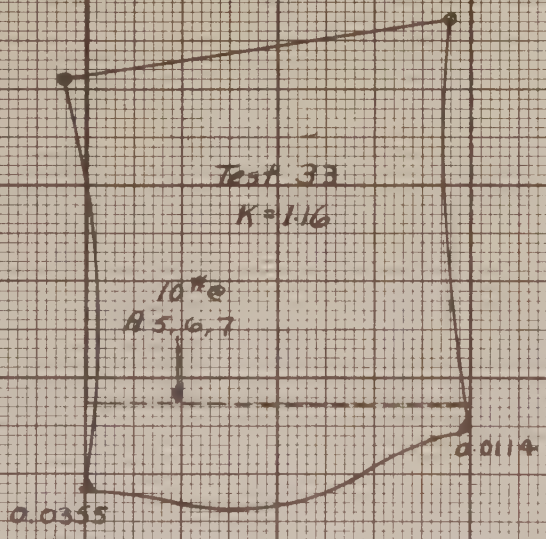
Series (b)





Deflections at Frame 5

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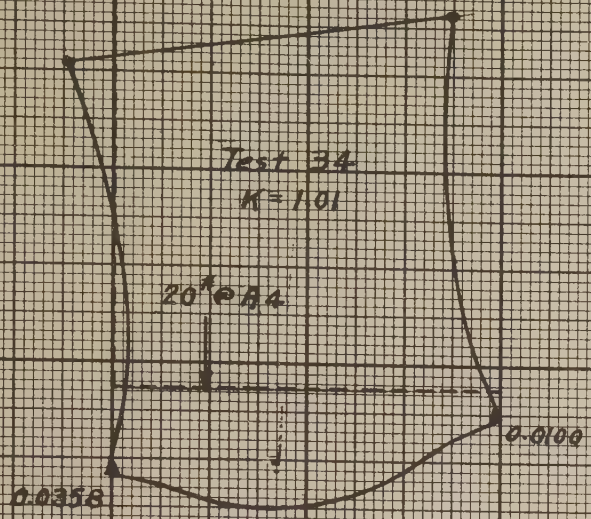
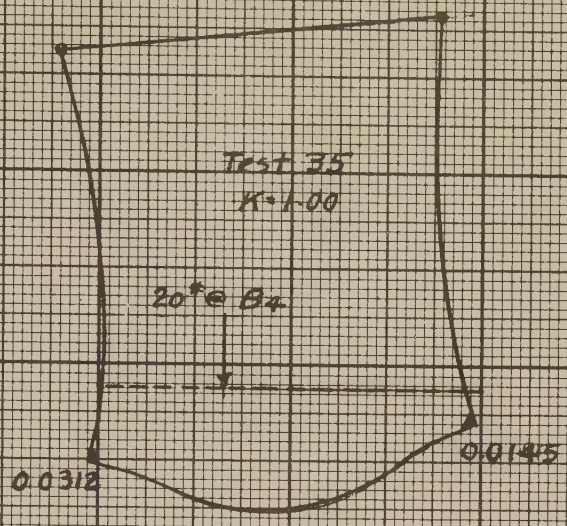
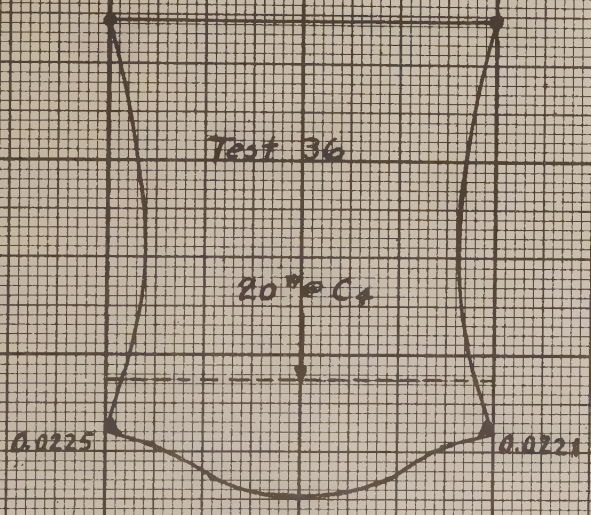
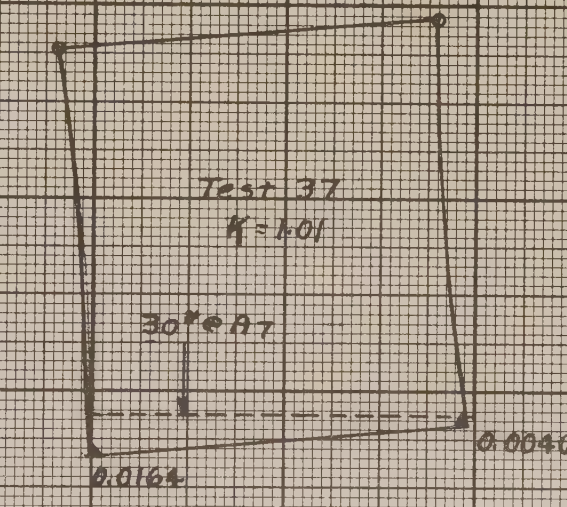
Series (c)









Deflection at Frame 4

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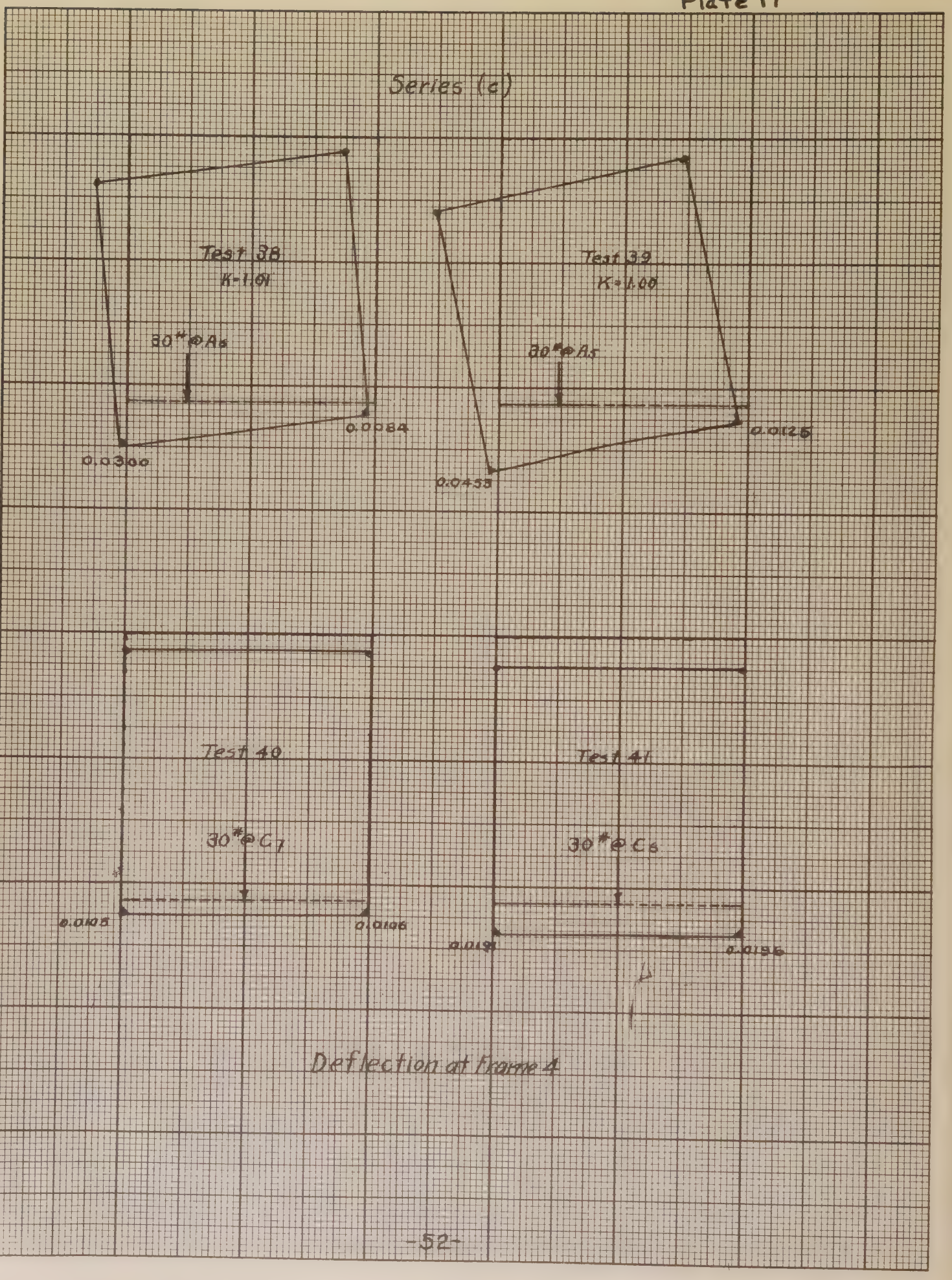

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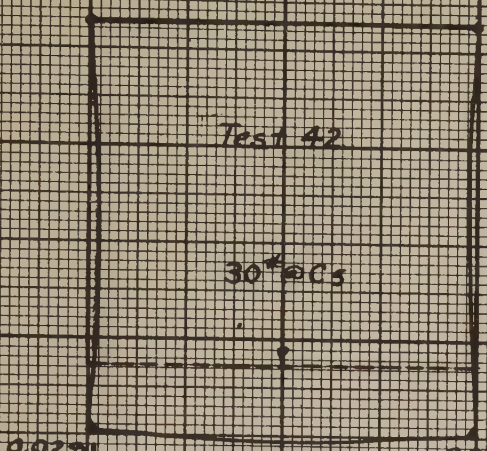
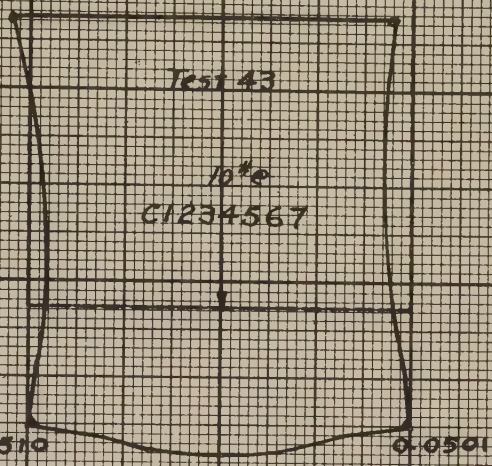
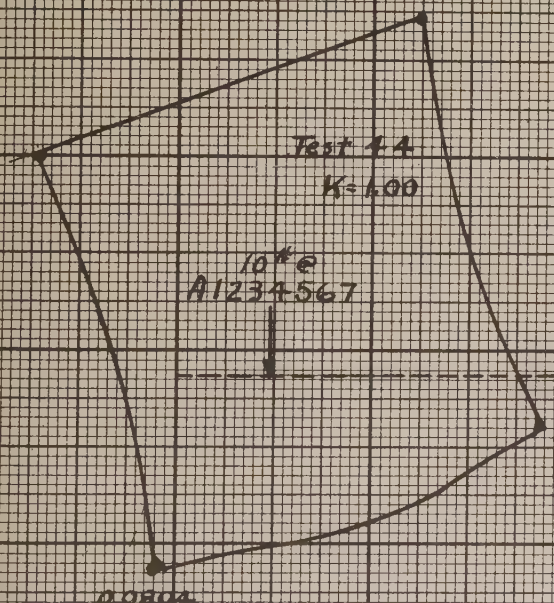
Series (c)



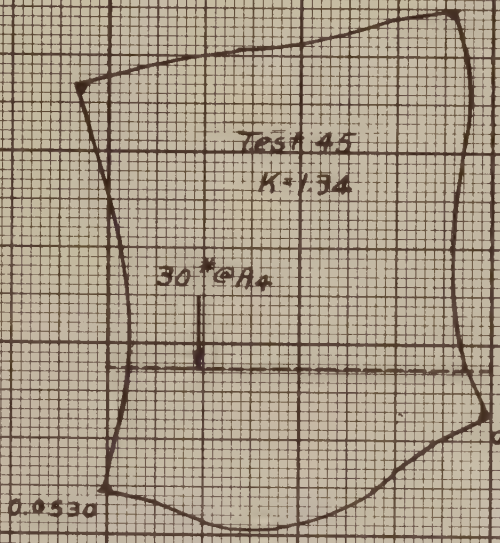


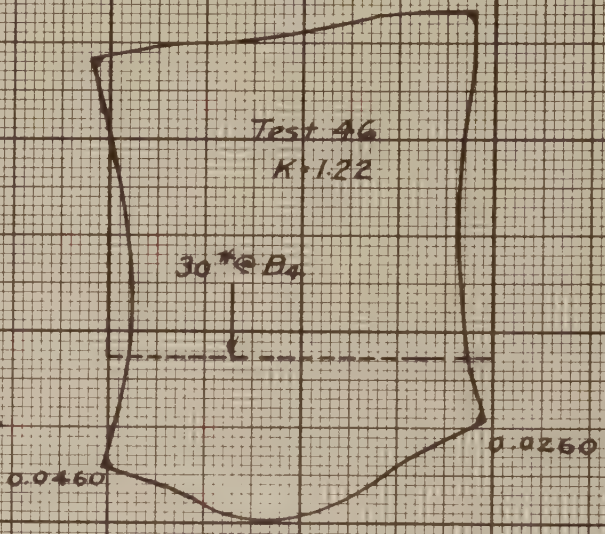


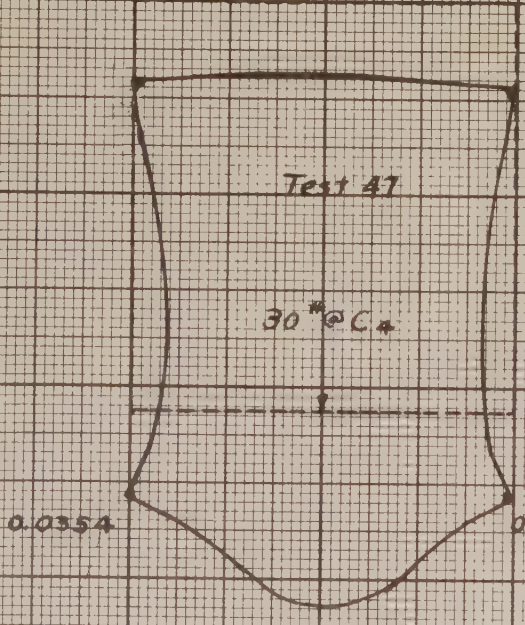
Deflection at Frame 4

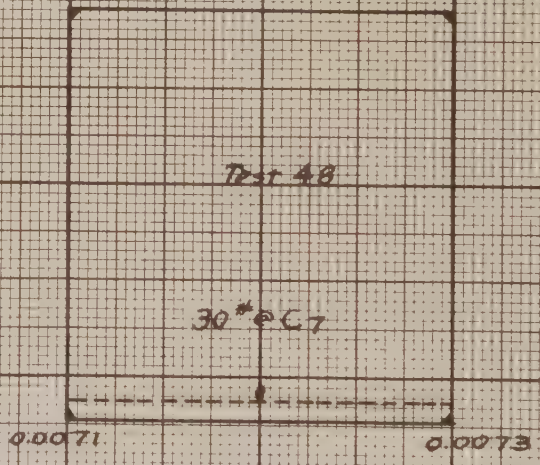
Plate 19

Series (d)









Deflection at Frame 4

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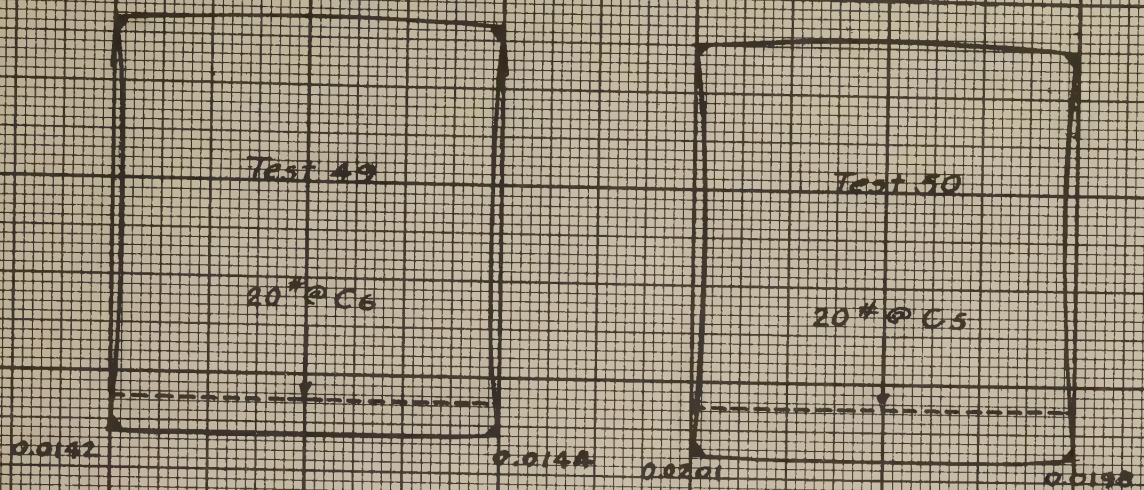
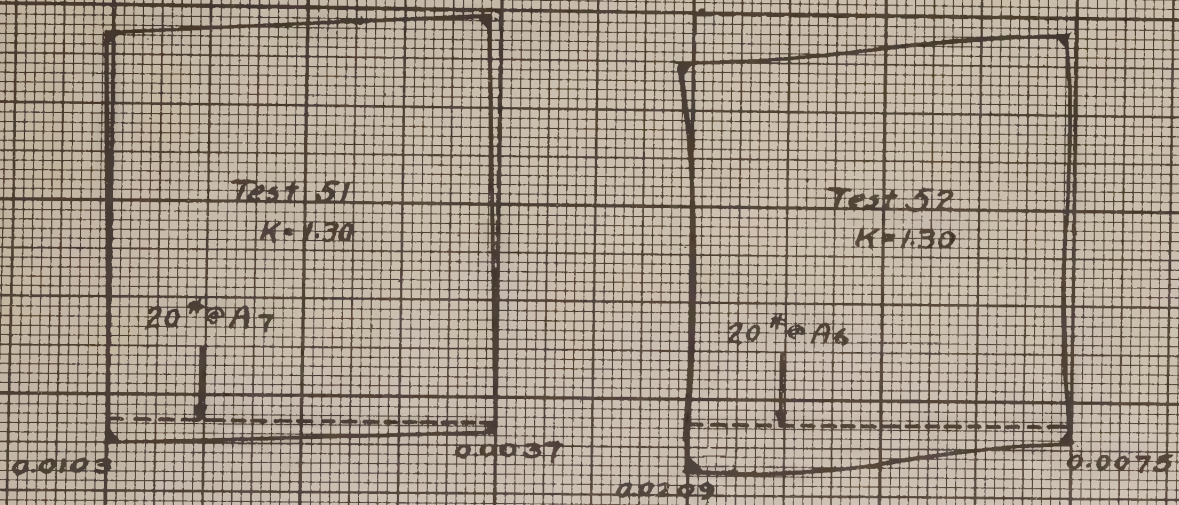
Series (d)





Deflection at Frame 4

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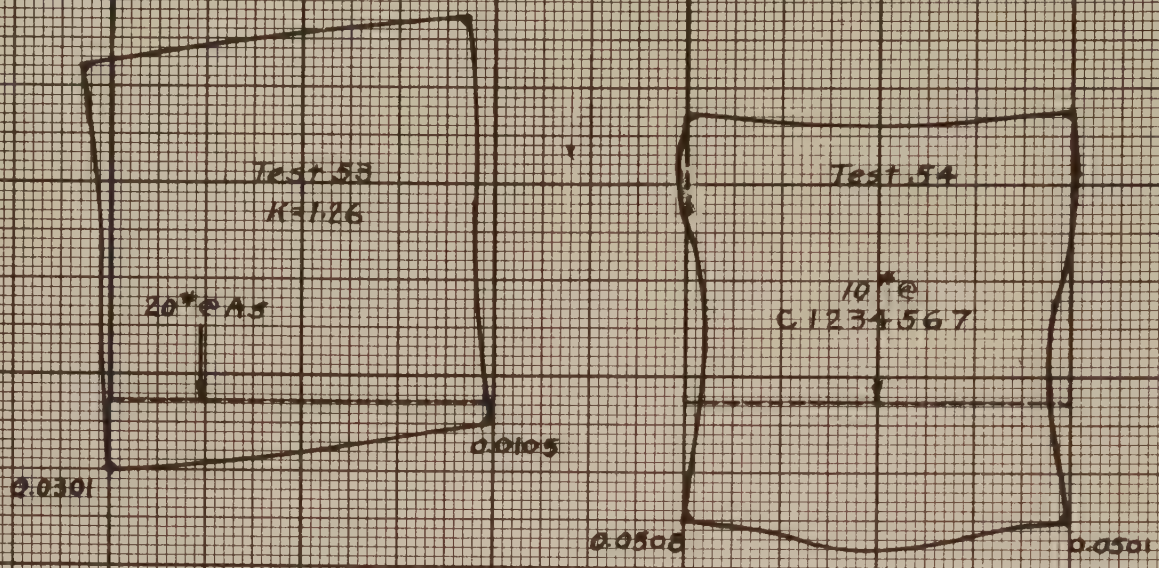
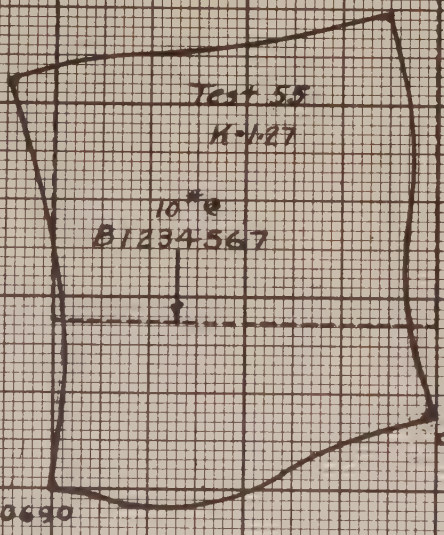
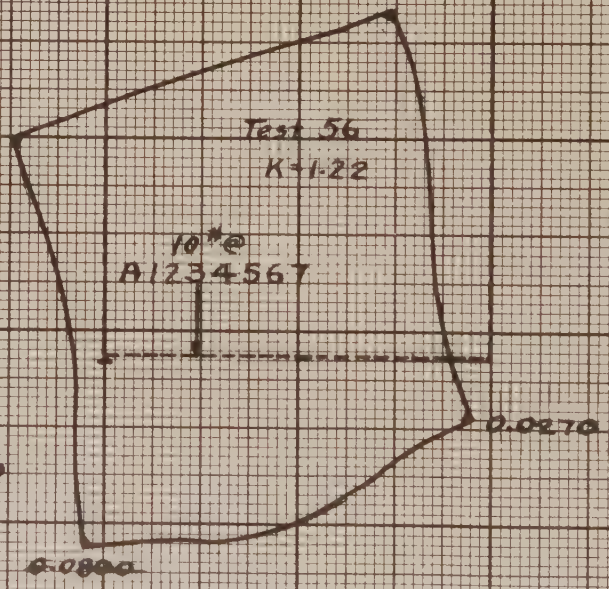
Series (d)







Deflection at Frame 4

SECTION VIII

Discussion

An examination of the k values found for the various tests leads to five specific conclusions:

(1) k is approximately 1.00 in a bridge with no upper wind bracing system (for example, in a pony truss). In series (c) (see Section VII) the k values for the model range from 1.00 to 1.01. The slight degree of torsional stiffness indicated by the value 1.01 is probably due to restraint of the chords against torsion and bending; and this may be expected to occur in an actual bridge.

(2) With simple pinned ties between opposite upper panel points, and with upper horizontal truss (series b), k values vary between 1.06 and 1.16 with an average of 1.11 for "A" loadings and between 1.08 and 1.09 for "B" loadings. It is apparent from Section VI that the factor for "A" loadings should be more than for "B" loadings and this is substantiated. It will be noted here that the greatest deviation from the average is only 4.5%. This error can be attributed to errors in test observations.

For exact agreement with the theoretical

values derived in Section VI, these values should be 1.06 and 1.03. The most probable source of this slight quantitative error is the discrepancy in the behavior of any structure with assumed theoretical behavior. Since the error is practically constant in magnitude and direction for all tests, it is proposed that a correction factor of 5% may be added to theoretically derived k values.

(3) With increasing fixity of the joints at the connections of the upper ties to the upper panel points (series a), the k factor increases. Series (a) shows average values of 1.19 for "A" loadings and 1.155 for "B" loadings. The increase of these values over the values found in series (b), as well as the further increase in the values found in series (d) as shown below, indicates that the torsional resistance of the bridge may be increased by strengthening the upper transverse ties.

(4) Series (d) shows k values ranging from 1.22 to 1.34 with an average of 1.28 for "A" loadings and 1.22 to 1.27 with an average of 1.245 for "B" loadings.

(5) It is important to note that the k values for the bridge for any specific test series are practically constant regardless of which frame is measured or where the loading is applied.

The writer realizes that it would aid the accuracy of test results to a considerable degree if the model were larger and if the deflections were of greater magnitude. Had time allowed, attainment of this end would be a second objective. It can be seen that with deflections in the magnitude of tenths of an inch instead of hundredths or thousandths, deflections can be measured far more accurately and slight errors will not affect the results.

Another desirable test which time does not allow, would be the experimental determination of k for varying span lengths. Reference to formula (16A) shows that the effect of m, the number of frames, is to make X_a greater, thus increasing k. Since this can be shown experimentally, it is recommended that future model studies be directed toward such demonstration.

As suggested in the early part of this paper, the study of torsional rigidity of ferry bridges, using the model herein described, should provide an interesting research subject.

The principal objectives of this investigation are believed to have been accomplished by the demonstration of the agreement between the calculated and experimental values of k. In no sense can this paper be considered to have completely covered the subject, but rather, to have but opened it for further investigation. Comparison of k determined in these tests with those computed for the field deflection values in Section III show some tendencies toward compatability. For example, it will be noted that the k value computed for bridges numbers 6 and 7 are 1.08 and 1.02 respectively. These bridges were 150 and 148 feet long. The length of the prototype of the celluloid model was

about 164 feet. Bridge number 6 had shallow upper cross bracing, fixed ended, and can be considered generally similar to the model in series (a) wherein the k values ranged between 1.15 and 1.19. The k for the model in series (b) was between 1.06 and 1.16 and the model in this case was more similar to bridge number 7 (k =1.02). It is not intended that these comparisons be strictly made, but they serve as indications of the agreement which can be expected by more rigorous similitude of model and prototype.

ACKNOWLEDGMENTS

The suggestions, criticisms and advice of Professor F. P. Witmer, Dr. R. K. Bernhard, and Mr. N. S. Glassman are acknowledged with appreciation.

The assistance of Mr. E. J. Glassman in performing the tests on the model is also appreciated.